

Standard 1: Number and Computation

FOURTH GRADE

Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 1: Number Sense – The student demonstrates number sense for whole numbers, fractions (including mixed numbers), decimals, and money including the use of concrete objects in a variety of situations.

Fourth Grade Knowledge Base Indicators	Fourth Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. knows, explains, and uses equivalent representations for (\\$): <ol style="list-style-type: none"> a. whole numbers from 0 through 100,000 (2.4.K1a-b); b. fractions greater than or equal to zero (halves, fourths, thirds, eighths, tenths, twelfths, sixteenths, hundredths) including mixed numbers (2.4.K1c); c. decimals greater than or equal to zero through hundredths place and when used as monetary amounts (2.4.K1c-d) (\\$), e.g., $7\text{¢} = \\$.07 = 7/100$ of a dollar or a hundreds grid with 7 sections colored or $.1 = 1/10 = \square\square\square\square\square\square\square\square$. 2. compares and orders: <ol style="list-style-type: none"> a. whole numbers from 0 through 100,000 (2.4.K1a-b) (\\$); b. fractions greater than or equal to zero (halves, fourths, thirds, eighths, tenths, twelfths, sixteenths, hundredths) including mixed numbers with a special emphasis on concrete objects (2.4.K1c); c. decimals greater than or equal to zero through hundredths place and when used as monetary amounts (2.4.K1c-d) (\\$). 	<p>The student...</p> <ol style="list-style-type: none"> 1. solves real-world problems using equivalent representations and concrete objects to (\\$): <ol style="list-style-type: none"> a. compare and order whole numbers from 0 through 100,000 (2.4.A1a-b); e.g., using base ten blocks, represent the attendance at the circus over a three day stay; then represent the numbers using digits and compare and order in different ways; b. add and subtract whole numbers from 0 through 10,000 and decimals when used as monetary amounts (2.4.A1a-d), e.g., use real money to show at least 2 ways to represent \$142.78, then subtract the cost of a pair of tennis shoes; c. multiply a one-digit whole number by a two-digit whole number (2.4.A1a-b), e.g., use base ten blocks to represent 24×5 to find the total number of hours in 5 days, or use repeated addition $24 + 24 + 24 + 24 + 25$ to solve, or use the algorithm. 2. determines whether or not solutions to real-world problems that involve the following are reasonable (\\$): <ol style="list-style-type: none"> a. whole numbers from 0 through 10,000 (2.4.A1a-b), e.g., a student says that there are 1,000 students in grade 4 at her school, is this reasonable? b. fractions greater than or equal to zero (halves, fourths, thirds, eighths, tenths, sixteenths) (2.4.A1c), e.g., you ate $\frac{1}{2}$ of a sandwich and a friend ate $\frac{3}{4}$ of the same sandwich; is this reasonable? c. decimals greater than or equal to zero when used as monetary amounts (2.4.A1c-d), e.g., a pack of chewing gum costs what amount - \$62 \$.75 9¢ 75.00 750¢? Is this reasonable?

Teacher Notes: Number sense refers to one’s ability to reason with numbers and to work with numbers in a flexible way. The ability to compute mentally, to estimate based on understanding of number relationships and magnitudes, and to judge reasonableness of answers are all involved in number sense.

When we say that someone has good number sense, we mean that he or she possesses a variety of abilities and understandings that include an awareness of the relationships between numbers, an ability to represent numbers in a variety of ways, a knowledge of the effects of operations, and an ability to interpret and use numbers in real-world counting and measurement situations. Such a person predicts with some accuracy the result of an operation and consistently chooses appropriate measurement units. This “friendliness with numbers” goes far beyond mere memorization of computational algorithms and number facts; it implies an ability to use numbers flexibly, to choose the most appropriate representation of a number for a given circumstance, and to recognize when operations have been correctly performed. (Number Sense and Operations: Addenda Series, Grades K-6, NCTM, 1993)

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$) . The National Standards in Personal Finance are included in the Appendix.

Standard 1: Number and Computation

FOURTH GRADE

Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 2: Number Systems and Their Properties – The student demonstrates an understanding of whole numbers with a special emphasis on place value; recognizes, uses, and explains the concepts of properties as they relate to whole numbers; and extends these properties to fractions (including mixed numbers), decimals, and money.

Fourth Grade Knowledge Base Indicators	Fourth Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. ▲ identifies, models, reads, and writes numbers using numerals, words, and expanded notation from hundredths place through one-hundred thousands place (2.4.K1a-b) (\$), e.g., four hundred sixty-two thousand, two hundred eighty-four and fifty hundredths = 462,284.50 or $462,284.50 = (4 \times 100,000) + (6 \times 10,000) + (2 \times 1,000) + (2 \times 100) + (8 \times 10) + (4 \times 1) + (5 \times .1) + (0 \times .01) = 400,000 + 60,000 + 2,000 + 200 + 80 + 4 + .5 + .00$. 2. classifies various subsets of numbers as whole numbers, fractions (including mixed numbers), or decimals (2.4.K1b-c, 2.4.K1i). 3. identifies the place value of various digits from hundredths place through one hundred thousands place (2.4.K1b) (\$). 4. identifies any whole number as even or odd (2.4.K1a). 5. uses the concepts of these properties with the whole number system and demonstrates their meaning including the use of concrete objects (2.4.K1a) (\$): <ol style="list-style-type: none"> a. ▲ commutative properties of addition and multiplication, e.g., $12 + 18 = 18 + 12$ and $8 \times 9 = 9 \times 8$; b. ▲ zero property of addition (additive identity) and property of one for multiplication (multiplicative identity), e.g., $24 + 0 = 24$ and $75 \times 1 = 75$; c. ▲ associative properties of addition and multiplication, e.g., $4 + (2 + 3) = (4 + 2) + 3$ and $2 \times (3 \times 4) = (2 \times 3) \times 4$; d. ▲ symmetric property of equality applied to addition and multiplication, e.g., $100 = 20 + 80$ is the same as $20 + 80 = 100$ and $21 = 7 \times 3$ is the same as $3 \times 7 = 21$; e. zero property of multiplication, e.g., $9 \times 0 = 0$ or $0 \times 112 = 0$; 	<p>The student...</p> <ol style="list-style-type: none"> 1. solves real-world problems with whole numbers from 0 through 10,000 using place value models; money; and the concepts of these properties to explain reasoning (2.4.A1a-b,d) (\$): <ol style="list-style-type: none"> a. commutative properties of addition and multiplication, e.g., a student has a \$5, a \$10, and a \$20 bill; a student totals the amount to see how much can be spent shopping for school supplies. The student says: Because you can add in any order, I can rearrange the money and count \$20, \$10, and \$5 for $\\$20 + \\$10 + \\$5$. Another student has 4 \$5 bills. The student is asked the amount. The student says: I don't know 4×5 but I know 5×4 is \$20, since multiplication can be done in any order. b. zero property of addition, e.g., a student has 6 marbles in one pocket and none in the other pocket. How many marbles altogether? c. property of one for multiplication, e.g., there are 24 students in our class, each student should have one math book; so I compute $24 \times 1 = 24$. Multiplying times 1 does not change the product because it is one group of 24. d. associative properties of addition and multiplication, e.g., a student has two dimes and a quarter. Using coins or money models, there are at least 2 ways to group the coins to find the total. One way is $10\text{¢} (\text{dime}) + 10\text{¢} (\text{dime}) = 20\text{¢}$, then add the quarter, so $20\text{¢} + 25\text{¢} (\text{quarter}) = 45\text{¢}$. Another way $10\text{¢} (\text{dime}) + 25\text{¢} (\text{quarter}) = 35\text{¢}$, then add the other dime to 35¢ so $35\text{¢} + 10\text{¢} = 45\text{¢}$. This models that $(D + D) + Q = D + (D + Q)$.

- ▲ – Assessed Indicator
- – Assessed Indicator on the Optional Constructed Response Assessment
- N – Noncalculator
- (**\$**) – Financial Literacy

THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.

<p>f. distributive property, e.g., $6(7 + 3) = (6 \cdot 7) + (6 \cdot 3)$.</p>	<p>e. zero property of multiplication, e.g., in science, you are observing a snail. The snail does not move over a 4-hour period. To figure its total movement, you say $4 \times 0 = 0$.</p> <p>2. performs various computational procedures with whole numbers from 0 through 10,000 using the concepts of the following properties; extends the properties to fractions (halves, fourths, thirds, eighths, tenths, sixteenths) including mixed numbers, and decimals through hundredths place; and explains how the properties were used (2.4.A1a-c):</p> <p>a. commutative property of addition and multiplication, e.g., $5 + 6 = 6 + 5$, the student says: I know that $5 + 6 = 11$ and adding in any order still gets the answer, so $6 + 5$ is the same as $5 + 6$. $4 \times 6 = 6 \times 4$, the student says: I know that $4 \times 6 = 24$ and multiplying in any order still gets the answer, so 4×6 is the same as 6×4.</p> <p>b. zero property of multiplication without computing, e.g., $158 \times 0 = 0$; the student says: I know the answer (product) is zero because no matter how many factors you have, when you multiply with a 0, the product is zero.</p> <p>c. associative property of addition, e.g., $9 + 8$ could be solved as $1 + (8 + 8)$ or $(1 + 8) + 8$, the student says: I don't know $9 + 8$, but I know my doubles of $8 + 8$, so I made the 9 into $1 + 8$ and then added 1 more to make 17.</p> <p>3. states the reason for using whole numbers, fractions, mixed numbers, or decimals when solving a given real-world problem (2.4.A1a-d).</p>
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Teacher Notes: From the Mathematics Dictionary and Handbook (Nichols Schwartz Publishing, 1999), **property** as a mathematical term means a characteristic (an attribute) of a number, geometric shape, mathematical operation, equation, or inequality. To give an example:

Property of a number: 8 is divisible by 2.

Property of a geometric shape: Each of the four sides of a square is of the same length.

Property of an operation: Addition is commutative. For all numbers x and y , $x + y = y + x$.

Property of an equation: For all numbers a , b , and c , if $a = b$, then $a + c = b + c$.

Property of an inequality: For all numbers a , b , and c , if $a > b$, then $a - c > b - c$.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (**\$**). The National Standards in Personal Finance are included in the Appendix.

Standard 1: Number and Computation

FOURTH GRADE

Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 3: Estimation – The student uses computational estimation with whole numbers, fractions (including mixed numbers) and money in a variety of situations.

Fourth Grade Knowledge Base Indicators	Fourth Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> estimates whole number quantities from 0 through 10,000; fractions (halves, fourths, thirds); and monetary amounts through \$1,000 using various computational methods including mental math, paper and pencil, concrete materials, and appropriate technology (2.4.K1a-d) (\$). uses various estimation strategies and explains how they are used when estimating whole numbers quantities from 0 through 10,000; fractions [(halves, fourths, thirds) including mixed numbers]; and monetary amounts through \$1,000 (2.4.K1a-d) (\$). recognizes and explains the difference between an exact and an approximate answer (2.4.K1a), e.g., when asked how many desks are in the room, the student gives an estimate of about 30 and then counts the desks and indicates an exact answer is 28 desks. selects from an appropriate range of estimation strategies and determines if the estimate is an overestimate or underestimate, (2.4.K1a). 	<p>The student...</p> <ol style="list-style-type: none"> adjusts original whole number estimates of a real-world problem using numbers from 0 through 10,000 based on additional information (a frame of reference) (2.4.A1a) (\$), e.g., if given a small jar and told the number of pieces of candy it has in it, the student would adjust his/her original estimate of the number of pieces of candy in a larger jar. estimates to check whether or not the result of a real-world problem using whole numbers from 0 through 10,000, fractions (including mixed numbers), and monetary amounts is reasonable and makes predictions based on the information (2.4.A1a-d) (\$), e.g., at the movies, you bought popcorn for \$2.35, a soda for \$2.50, and paid \$4.50 for the ticket. Is it reasonable to say you spent \$10? How much will you need to save to go to the movies once a week for the next month? selects a reasonable magnitude from three given quantities based on a familiar problem situation and explains the reasonableness of selection (2.4.A1a), e.g., about how many new pencils will fit in your pencil box? Is it about 25, about 50, or about 100? The answer will depend on the size of your pencil box. determines if a real-world problem calls for an exact or approximate answer and performs the appropriate computation using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.A1a) (\$).

Teacher Notes: Estimate, as a verb, means to make an educated guess based on information in a problem or to give an answer close to the exact number. Estimation is used when an exact answer is not needed, as in many real-life situations for which “ballpark” computations are acceptable. Good number sense enables one to estimate a quantity, estimate a measure, or estimate an answer.

Estimation serves as an important companion to computation. It provides a tool for judging the reasonableness of computational methods including mental math, paper and pencil, concrete objects, and appropriate technology. However, being able to compute does not automatically lead to an ability to estimate or judge reasonableness of answers. Frequent modeling by the teacher helps students develop a range of estimation strategies. Students should be encouraged to frequently explain their thinking as they estimate. As with exact computation, sharing estimation strategies allows students access to others’ thinking and provides opportunities for class discussion. Identifying the estimation strategy by name is not critical; however, explaining the thinking behind the strategy to make a valid estimation is important. (Principles and Standards for School Mathematics, NCTM, 2000).

Mental math and **estimation** are distinct but related mathematical skills. Proficiency in mental math contributes to increased skill in estimation. In order for students to become more familiar with estimation, teachers should introduce estimation with examples where rounded or estimated numbers are used. Emphasis should be placed on real-world examples where only estimation is required, e.g., About how many hours do you sleep a night? Using the language of estimation is important, so students begin to realize that a variety of estimates (answers) are possible. In addition, when students are taught specific estimation strategies, they develop mental math and estimation skills easier. Estimation strategies include front-end with adjustment, compatible “nice” numbers, clustering, or special numbers.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Constructed Response Assessment

N – Noncalculator

(\$) – Financial Literacy

THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.

Standard 1: Number and Computation

FOURTH GRADE

Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 4: Computation – The student models, performs, and explains computation with whole numbers, fractions, and money including the use of concrete objects in a variety of situations.

Fourth Grade Knowledge Base Indicators	Fourth Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. computes with efficiency and accuracy using various computational methods including mental math, paper and pencil, concrete materials, and appropriate technology (2.4.K1a) (\$). 2. N states and uses with efficiency and accuracy multiplication facts from 1 x 1 through 12 x 12 and corresponding division facts (2.4.K1a) (\$). 3. N performs and explains these computational procedures (\$): <ol style="list-style-type: none"> a. adds and subtracts whole numbers from 0 through 100,000 and when used as monetary amounts (2.4.K1a-b,d); b. multiplies through a three-digit whole number by a two-digit whole number (2.4.K1a-b); c. multiplies whole dollar monetary amounts (through three-digits) by a one- or two-digit whole number (2.4.K1d), e.g., \$45 x 16; d. multiplies monetary amounts less than \$100.00 by whole numbers less than ten (2.4.K1d), e.g., \$14.12 x 7; e. divides through a two-digit whole number by a one-digit whole number with a one-digit whole number quotient with or without a remainder (2.4.K1a-b), e.g., $47 \div 5 = 9 \text{ r } 2$; f. adds and subtracts fractions greater than or equal to zero with like denominators (2.4.K1c); g. figures correct change through \$20.00 (2.4.K1d). 4. identifies multiplication and division fact families (2.4.K1a). 5. reads and writes horizontally, vertically, and with different operational symbols the same addition, subtraction, multiplication, or division expression, e.g., $6 \cdot 4$ is the same as 6×4 is the same as 4 and $6(4)$ or 10 divided by 2 is the same as $10 \div 2$ or $\underline{10}$. x <u>6</u> <p style="text-align: center;">2</p>	<p>The student...</p> <ol style="list-style-type: none"> 1. ▲N solves one- and two-step real-world problems with one or two operations using these computational procedures (\$): <ol style="list-style-type: none"> a. adds and subtracts whole numbers from 0 through 10,000 and when used as monetary amounts (2.4.A1a-b,d), e.g., Lee buys a bicycle for \$139, a helmet for \$29, and a reflector for \$6. He paid for it with a \$200 check from his grandparents. How much will he have left from the \$200 check? b. multiplies through a two-digit whole number by a two-digit whole number (2.4.A1a-b), e.g., at school, there are 22 students in each classroom. If there are 24 classes, how many students are in the classrooms? c. multiplies whole dollar monetary amounts (up through three-digit) by a one- or two-digit whole number (2.4.A1a-b,d), e.g., 112 third and fourth graders are planning a field trip. The cost per student is \$9.00. How much will the trip cost? d. multiplies monetary amounts less than \$100 by whole numbers less than ten (2.4.A1a-d), e.g., at the book fair, a student buys 8 books on animals for \$2.69 each. How much did the student pay for the books? e. ■ figures correct change through \$20.00 (2.4.A1a-d), e.g., buying a 65¢ drink, paying for it with a \$1 bill, and then figuring the amount of change. 2. generates a family of multiplication and division facts given one equation/fact (2.4.A1b), e.g., given $8 \times 9 = 72$, the other facts are $9 \times 8 = 72$, $72 \div 8 = 9$, and $72 \div 9 = 8$.

- ▲ – Assessed Indicator**
- – Assessed Indicator on the Optional Constructed Response Assessment**
- N – Noncalculator**
- (\$) – Financial Literacy**

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| <p>6. ▲N shows the relationship between these operations with the basic fact families (addition facts with sums from 0 through 20 and corresponding subtraction facts, multiplication facts from 1 x 1 through 12 x 12 and corresponding division facts) including the use of mathematical models (2.4.K1a) (\$):</p> <ul style="list-style-type: none">a. addition and subtraction,b. addition and multiplication,c. multiplication and division,d. subtraction and division. <p>7. finds factors and multiples of whole numbers from 1 through 100 (2.4.K1a).</p> | |
|--|--|

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Constructed Response Assessment

N – Noncalculator

(\$) – Financial Literacy

THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.

Teacher Notes: Efficiency and accuracy means that students are able to compute single-digit numbers with fluency. Students increase their understanding and skill in single-digit addition and subtraction by developing relationships within addition and subtraction combinations and by counting on for addition and counting up for subtraction and unknown-addend situations. Students learn basic number combinations and develop strategies for computing that makes sense to them. Through class discussions, students can compare the ease of use and ease of explanation of various strategies. In some cases, their strategies for computing will be close to conventional algorithms; in other cases, they will be quite different. Many times, students' invented approaches are based on a sound understanding of numbers and operations, and these invented approaches often can be used with efficiency and accuracy. (Principles and Standards for School Mathematics, NCTM, 2000)

The definition of computation is finding the standard representation for a number. For example, $6 + 6$, 4×3 , $17 - 5$, and $24 \div 2$ are all representations for the standard representation of 12. **Mental math** is mentally finding the standard representation for a number – calculating in your head instead of calculating using paper and pencil or technology. One of the main reasons for teaching mental math is to help students determine if a computed/calculated answer is reasonable; in other words, using mental math to estimate to see if the answer makes sense. Students develop mental math skills easier when they are taught specific strategies. Mental math strategies include counting on, doubling, repeated doubling, halving, making tens, dividing with tens, thinking money, and using compatible “nice” numbers.

Regrouping refers to the reorganization of objects. In computation, **regrouping** is based on a “partitioning to multiples of ten” strategy. For example, $46 + 7$ could be solved by partitioning 46 into 40 and 6, then $40 + (6 + 7) = 40 + 13$ (and then 13 is partitioned into 10 and 3) which then becomes $(40 + 10) + 3$ becomes $50 + 3 = 53$ or 7 could be partitioned as 4 and 3, then $46 + 4$ (bridging through 10) = 50 and $50 + 3 = 53$. Before algorithmic procedures are taught, an understanding of “what happens” must occur. For this to occur, instruction should involve the use of structured manipulatives. To emphasize the role of the base ten numeration system in algorithms, some form of expanded notation is recommended. During instruction, each child should have a set of manipulatives to work with rather than sit and watch demonstrations by the teacher. Some additional computational strategies include *doubles plus one or two* (i.e., $6 + 8$, 6 and 6 are 12, so the answer must be 2 more or 14), *compensation* (i.e., $9 + 7$, if one is taken away from 9, it leaves 8; then that one is given to 7 to make 8, then $8 + 8 = 16$), *subtracting through ten* (i.e., $13 - 5$, 13 take away 3 is 10, then take 2 more away from 10 and that is 8), and *nine is one less than ten* (i.e., $9 + 6$, 10 and 6 are 16, and 1 less than 16 makes 15). (Teaching Mathematics in Grades K-8: Research Based Methods, ed. Thomas R. Post, Allyn and Bacon, 1988)

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

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Standard 2: Algebra

FOURTH GRADE

4-10
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Constructed Response Assessment

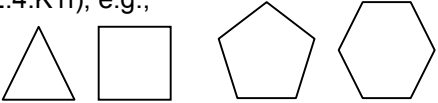
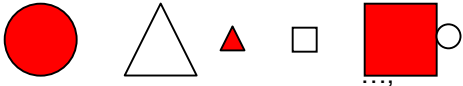
N – Noncalculator

(\$) – Financial Literacy

THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 1: Patterns – The student recognizes, describes, extends, develops, and explains relationships in patterns using concrete objects in a variety of situations.

Fourth Grade Knowledge Base Indicators	Fourth Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> uses concrete objects, drawings, and other representations to work with types of patterns(2.4.K1a): <ol style="list-style-type: none"> repeating patterns, e.g., an AB pattern is like 1-2, 1-2, ...; an ABC pattern is like dog-horse-pig, dog-horse-pig, ...; an AAB pattern is like $\uparrow\uparrow\rightarrow$, $\uparrow\uparrow\rightarrow$, ...; growing patterns e.g., 2, 5, 11, 20, ... uses these attributes to generate patterns: <ol style="list-style-type: none"> counting numbers related to number theory (2.4.K1a), e.g., multiples and factors through 12 or multiplying by 10, 100, or 1,000; whole numbers that increase or decrease (2.4.K1a) (\$), e.g., 20, 15, 10, ...; geometric shapes including one or two attributes changes (2.4.K1f), e.g., <div style="text-align: center;">  <p>... when the next shape has one more side; or when both color and shape change at the same time such as</p>  </div> measurements (2.4.K1a), e.g., 3 ft., 6 ft., 9 ft., ...; money and time (2.4.K1a,d) (\$), e.g., \$.25, \$.50, \$.75, ... or 1:05 p.m., 1:10 p.m., 1:15 p.m., ...; things related to daily life (2.4.K1a), e.g., water cycle, food cycle, or life cycle; things related to size, shape, color, texture, or movement (2.4.K1a), e.g., rough, smooth, rough, smooth, rough, smooth, or clapping hands (kinesthetic patterns). 	<p>The student...</p> <ol style="list-style-type: none"> generalizes these patterns using a written description: <ol style="list-style-type: none"> counting numbers related to number theory (2.4.A1a), whole number patterns (2.4.A1a) (\$), patterns using geometric shapes (2.4.A1f), measurement patterns (2.4.A1a), money and time patterns (2.4.A1a,d) (\$), patterns using size, shape, color, texture, or movement (2.4.A1a). recognizes multiple representations of the same pattern (2.4.A1a), e.g., skip counting by 5s to 60; whole number multiples of 5 through 60; the multiplication table of 5 given the numerical pattern of 5, 10, 15, ..., 60; relating the concept of five minute time intervals to each of the numerals on a clock giving the pattern of 5, 10, 15, ..., 60; one nickel, two nickels, three nickels, ...; the number of fingers on twelve hands; recognizing that all of these representations are the same general pattern.

3. identifies, states and continues a pattern presented in visual various formats including numeric (list or table), visual (picture, table, or graph), verbal (oral description), kinesthetic (action), and written (2.4.K1a) (**\$**).
4. generates:
 - a. a pattern (repeating, growing) (2.4.K1a);
a pattern using a function table (input/output machines, T-tables) (2.4.K1e).

Teacher Notes: Working with **patterns** is an important process in the development of mathematical thinking. Patterns can be based on geometric attributes (shapes, regions, angles); measurement attributes (color, texture, length, weight, volume, number); relational attributes (proportion, sequence, functions); and affective attributes (values, likes, dislikes, familiarity, heritage, culture). (Learning to Teach Mathematics, Randall J. Souviney, Macmillan Publishing Company, 1994)

This process (working with patterns) can be used to develop or deepen understandings of important concepts in number theory, whole numbers, measurement, geometry, probability, and functions. Working with patterns provides opportunities for students to recognize, describe, extend, develop, and explain.

Number theory is the study of the properties of the counting numbers (positive integers), their relationships, ways to represent them, and patterns among them. Number theory includes the concepts of odd and even numbers, factors and multiples, primes and composites, and greatest common factor and least common multiple.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (**\$**). The National Standards in Personal Finance are included in the Appendix.

4-12
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Constructed Response Assessment

N – Noncalculator

(**\$**) – Financial Literacy

THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.

Standard 2: Algebra

FOURTH GRADE

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 2: Variables, Equations, and Inequalities – The student uses variables, symbols, and whole numbers to solve equations including the use of concrete objects in a variety of situations.

Fourth Grade Knowledge Base Indicators	Fourth Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. explains and uses variables and symbols to represent unknown whole number quantities from 0 through 1,000 (2.4.K1a). 2. ▲ solves one-step equations using whole numbers with one variable and a whole number solution that: <ol style="list-style-type: none"> a. find the unknown in a multiplication or division equation based on the multiplication facts from 1 x 1 through 12 x 12 and corresponding division facts (2.4.K1a), e.g., $60 = 10 \times n$; b. find the unknown in a money equation using multiplication and division based upon the facts and addition and subtraction with values through \$10 (2.4.K1d) (\$), e.g., 8 quarters + 10 dimes = y dollars; c. find the unknown in a time equation involving whole minutes, hours, days, and weeks with values through 200 (2.4.K1a), e.g., 180 minutes = y hours. 3. compares two whole numbers from 0 through 10,000 using the equality and inequality symbols (=, ≠, <, >) and their corresponding meanings (is equal to, is not equal to, is less than, is greater than) (2.4.K1b) (\$). 4. reads and writes whole number equations and inequalities using mathematical vocabulary and notation, e.g., $15 = 3 \times 5$ is the same as fifteen equals three times five or $4,564 > 1,000$ is the same as four thousand, five hundred sixty-four is greater than one thousand. 	<p>The student...</p> <ol style="list-style-type: none"> 1. represents real-world problems using variables and symbols with unknown whole number quantities from 0 through 1,000 (2.4.A1a) (\$), e.g., How many weeks in twenty-eight days? can be represented by $n \times 7 = 28$ or $n = 28 \div 7$. 2. generates one-step equations to solve real-world problems with one unknown (represented by a variable or symbol) and a whole number solution that (2.4.A1a) (\$): <ol style="list-style-type: none"> a. add or subtract whole numbers from 0 through 1,000; e.g., Homer, Kansas has 832 nonfiction books in its library. Homer, Idaho has 652 nonfiction books in its library. How many fewer books nonfiction books are in Homer, Idaho's library? $832 - 652 = B$; b. multiply or divide using the basic facts, e.g., Tom has a sticker book and each page holds 5 stickers. If the same number of stickers is placed on each page, the book will hold 30 stickers. How many pages are in his book? This is represented by $5 \times S = 30$ or $30 \div 5 = S$. 3. generates (2.4.A1a) (\$): <ol style="list-style-type: none"> a. real-world problems with one operation to match a given addition, subtraction, multiplication, or division equation using whole numbers through 99, e.g., given $12 \times 3 = Y$, the student writes: I was sick for 3 days, when I got back I had 3 pages of homework. There are 12 problems on each page. How many total problems must I work? b. number comparison statements using equality and inequality symbols (=, <, >) with whole numbers, measurement, and money, e.g., $1 \text{ ft} < 15 \text{ in}$ or $10 \text{ quarters} > \\$2$.

Teacher Notes: Understanding the **concept of variable** is fundamental to algebra. Students use various symbols, including letters and geometric shapes, to represent unknown quantities that both do and do not vary. Quantities that are not given and do not vary are often referred to as unknowns or missing elements when they appear in equations, e.g., $2 + 4 = \Delta$ or $3 + s = 7$. Various symbols or letters should be used interchangeably in equations.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.

4-14
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Constructed Response Assessment

N – Noncalculator

(\$) – Financial Literacy

THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.

Standard 2: Algebra

FOURTH GRADE

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 3: Functions – The student recognizes and describes whole number relationships including the use of concrete objects in a variety of situations.

Fourth Grade Knowledge Base Indicators	Fourth Grade Application Indicators																														
<p>The student...</p> <ol style="list-style-type: none"> states mathematical relationships between whole numbers from 0 through 1,000 using various methods including mental math, paper and pencil, concrete materials, and appropriate technology (2.4.K1a) (\$). \blacktriangle finds the values, determines the rule, and states the rule using symbolic notation with one operation of whole numbers from 0 through 200 using a horizontal or vertical function table (input/output machine, T-table) (2.4.K1e), e.g., using the function table, find the rule, the rule is $N \cdot 4$. <table border="1" data-bbox="512 711 716 935" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>N</th> <th>?</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>4</td> </tr> <tr> <td>5</td> <td>20</td> </tr> <tr> <td>2</td> <td>8</td> </tr> <tr> <td>3</td> <td>?</td> </tr> <tr> <td>4</td> <td>?</td> </tr> <tr> <td>?</td> <td>24</td> </tr> </tbody> </table> generalizes numerical patterns using whole numbers from 0 through 200 with one operation by stating the rule using words, e.g., if the pattern is 46, 68, 90, 112, 134, ...; in words, the rule is add 22 to the number before. uses a function table (input/output machine, T-table) to identify, plot, and label the ordered pairs in the first quadrant of a coordinate plane (2.4.K1a,e). 	N	?	1	4	5	20	2	8	3	?	4	?	?	24	<p>The student...</p> <ol style="list-style-type: none"> \blacktriangle represents and describes mathematical relationships between whole numbers from 0 through 1,000 using concrete objects, pictures, written descriptions, symbols, equations, tables, and graphs (2.4.A1a) (\$). finds the rule, states the rule, and extends numerical patterns using real-world applications using whole numbers from 0 through 200 (2.4.A1a,e), e.g., the teacher must order supplies for field day. For every 12 students, one red rubber ball is needed. If 6 balls are ordered, how many students will be able to play? A solution using a function table might be: <table border="1" data-bbox="1234 753 1755 1032" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Number of Students</th> <th>Number of Balls</th> </tr> </thead> <tbody> <tr> <td>12</td> <td>1</td> </tr> <tr> <td>24</td> <td>2</td> </tr> <tr> <td>36</td> <td>3</td> </tr> <tr> <td>48</td> <td>4</td> </tr> <tr> <td>60</td> <td>5</td> </tr> <tr> <td>72</td> <td>6</td> </tr> <tr> <td>N</td> <td>$N \div 12$</td> </tr> </tbody> </table> <p>The rule is divide the number of students by 12 or for each group of 12 students, another ball is added. Other solutions might be using a pattern to count by 12 six times – 12, 24, 36, 48, 60, 72 or to skip count by 12 for each ball ordered.</p>	Number of Students	Number of Balls	12	1	24	2	36	3	48	4	60	5	72	6	N	$N \div 12$
N	?																														
1	4																														
5	20																														
2	8																														
3	?																														
4	?																														
?	24																														
Number of Students	Number of Balls																														
12	1																														
24	2																														
36	3																														
48	4																														
60	5																														
72	6																														
N	$N \div 12$																														

Teacher Notes: Functions are relationships or rules in which each member of one set is paired with one, and only one, member of another set (an ordered pair). The concept of function can be introduced using function machines. Any number put in the machine will be changed according to some rule. A record of inputs and corresponding outputs can be maintained in a two-column format. Function tables, input/output machines, and T-tables may be used interchangeably and serve the same purpose.

Function concepts should be developed from **growing patterns**. Each term in a number sequence is related to its position in the sequence – the functional relationship. The pattern – 4, 7, 10, 13, 16, 19, and so on – is an arithmetic sequence *with a difference of 3*. The pattern could be described as *add 3* meaning that 3 must be added to the previous term to find the next. This pattern is explained by using the recursive definition for a sequence. The recursive definition for a sequence is a statement or a set of statements that explains how each successive term in the sequence is obtained from the previous term(s).

In the pattern 1, 4, 9, 16, 25, ..., 225; there is *no common difference*. This sequence is not arithmetic or geometric (no common ratio between geometric terms). Neither is it a combination of the two; however, there is a pattern and the missing terms between 25 and 225 can be found. To find the term value, square the number of the term. The next missing terms would be 36, 49, 64, 81, 100, 121, and 144. This pattern is explained by using the explicit formula for a sequence. The explicit formula for a sequence defines a rule for finding each term in the number sequence related to its position in the sequence. In other words, to find the term value, square the number of the term – the 5th term is 5², the 8th term is 8², ...

Patterns themselves are not explicit or recursive. The *RULE* for the pattern can be expressed explicitly or recursively and *MOST* patterns can be explained using either format especially *IF* that pattern reflects either an arithmetic sequence or geometric sequence.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (**\$**). The National Standards in Personal Finance are included in the Appendix.

4-16
January 31, 2004

▲ – Assessed Indicator

■ – Assessed Indicator on the Optional Constructed Response Assessment

N – Noncalculator

(**\$**) – Financial Literacy

THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.

Standard 2: Algebra

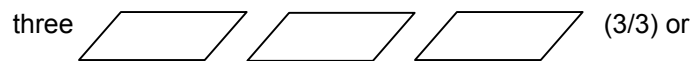
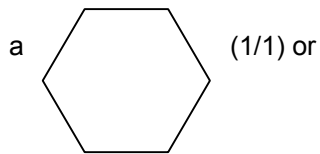
FOURTH GRADE

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 4: Models – The student develops and uses mathematical models including the use of concrete objects to represent and explain mathematical relationships in a variety of situations.

Fourth Grade Knowledge Base Indicators	Fourth Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. knows, explains, and uses mathematical models to represent mathematical concepts, procedures, and relationships. Mathematical models include: <ol style="list-style-type: none"> a. process models (concrete objects, pictures, diagrams, number lines, hundred charts, measurement tools, multiplication arrays, division sets, or coordinate planes/grids) to model computational procedures, mathematical relationships, and equations (1.1.K1a, 1.1.K2a, 1.2.K1, 1.2.K4-5, 1.3.K1-4, 1.4.K1-2, 1.4.K3a-b, 1.4.K3e, 1.4.K4, 1.4.K6-7, 2.1.K1, 2.1.K.1a-b, 2.1.K2d-g, 2.1.K3, 2.1.K4a, 2.2.K1, 2.2.K2a, 2.2.K3-4, 2.3.K1, 2.3.K4, 3.2.K1-4, 3.3.K1-2, 3.4.K1-4, 4.2.K3) (\$); b. place value models (place value mats, hundred charts, base ten blocks, or unifix cubes) to compare, order, and represent numerical quantities and to model computational procedures (1.1.K1a, 1.1.K2a, 1.2.K1-3, 1.3.K1-2, 1.4.K3a-b, 1.4.K3e, 2.2.K4) (\$); c. fraction and mixed number models (fraction strips or pattern blocks) and decimal models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.K1b-c, 1.1.K2b-c, 1.2.K2, 1.3.K1-2, 1.4.K1f) (\$); d. money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.K1c, 1.2.K1c, 1.3.K1-2, 1.4.K3a, 1.4.K3a, 1.4.K3c-d, 1.4.K3g, 2.1.K2e, 2.2.K2b) (\$); e. function tables (input/output machines, T-tables) to model numerical and algebraic relationships (2.1.K4b, 2.3.K2, 2.3.K4, 3.4.K4) (\$); 	<p>The student...</p> <ol style="list-style-type: none"> 1. recognizes that various mathematical models can be used to represent the same problem situation. Mathematical models include: <ol style="list-style-type: none"> a. process models (concrete objects, pictures, diagrams, number lines, coordinate planes/grids, hundred charts, measurement tools, multiplication arrays, or division sets) to model computational procedures, mathematical relationships, and problem situations (1.1.A1, 1.1.A2a, 1.2.A1-3, 1.3.A1-4, 1.4.A1, 2.1.A1a-b, 2.1.A1d-f, 2.1.A2, 2.2.A1-3, 2.3.A1-2, 3.2.A1a-g, 3.2.A2-3, 3.3.A1-2, 3.4.A1-2, 4.2.A2) (\$); b. place value models (place value mats, hundred charts, base ten blocks, or unifix cubes) to model problem situations (1.1A1, 1.1.A2a, 1.2.A1-3, 1.3.A2, 1.4.A1) (\$); c. fraction and mixed number models (fraction strips or pattern blocks) and decimal models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.A1b, 1.1.A2b-c, 1.2.A2-3, 1.3.A2, 1.4.A1d-e) (\$); d. money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.A1b, 1.1.A2c, 1.2.A1, 1.2.A3, 1.3.A1, 1.4.A1a, 1.4.A1c-e, 2.1.A1e) (\$); e. function tables (input/output machines, T-tables) to model numerical and algebraic relationships (2.3.A2) (\$); f. two-dimensional geometric models (geoboards, dot paper, pattern blocks, or tangrams) to model perimeter, area, and properties of geometric shapes and three-dimensional geometric models (solids) and real-world objects to compare size and to model properties of geometric shapes (2.1.A1c, 3.1.A1-2, 3.2.A1h, 3.3.A3);

- f. two-dimensional geometric models (geoboards, dot paper, pattern blocks, or tangrams) to model perimeter, area, and properties of geometric shapes and three-dimensional geometric models (solids) and real-world objects to compare size and to model properties of geometric shapes (2.1.K2c, 2.1.K1e, 3.1.K1-6, 3.2.K5, 3.3.K3);
 - g. two-dimensional geometric models (spinners), three-dimensional models (number cubes), and process models (concrete objects) to model probability (4.1.K1-3) (\$);
 - h. graphs using concrete objects, pictographs, frequency tables, horizontal and vertical bar graphs, line graphs, circle graphs, Venn diagrams, line plots, charts, and tables to organize and display data (4.1.K2, 4.2.K1-2) (\$);
 - i. Venn diagrams to sort data and show relationships (1.2.K2).
2. creates a mathematical model to show the relationship between two or more things, e.g., using pattern blocks, a whole (1) can be represented as



- g. two-dimensional geometric models (spinners), three-dimensional geometric models (number cubes), and process models (concrete objects) to model probability (4.1.A1-3) (\$);
 - h. graphs using concrete objects, pictographs, frequency tables, horizontal and vertical bar graphs, line graphs, Venn diagrams, line plots, charts, and tables to organize, display, explain, and interpret data (4.1.A2, 4.2.A1, 4.2.A3-4) (\$);
 - i. Venn diagrams to sort data and show relationships.
2. selects a mathematical model and explains why some mathematical models are more useful than other mathematical models in certain situations.

Teacher Notes: For assessment purposes, the mathematical modeling process appropriate to the indicator may be included as part of the item being assessed.

The **mathematical modeling** process involves:

- a. selecting key features and relationships within the real-world situation and representing these concepts in mathematical terms through some sort of mathematical model,
- b. performing manipulations and mathematical procedures within the mathematical model,
- c. interpreting the results of the manipulations within the mathematical model,
- d. using these results to make inferences about the original real-world situation.

The use of **mathematical models** is necessary for conceptual understanding. The ways in which mathematical ideas are represented is fundamental to how students understand and use those ideas. As students begin to use multiple representations of the same situation, students begin to develop an understanding of the advantages and disadvantages of the various representations/models.

Many **mathematical models** are listed in this benchmark. The indicator lists some of the mathematical models that could be used to teach a concept. Each indicator in this benchmark is linked to other indicators in other benchmarks; those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3. In addition, the indicator in the other benchmarks identifies, in parentheses, the Models' indicator. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models).

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a **(\$)**. The National Standards in Personal Finance are included in the Appendix.

Standard 3: Geometry

FOURTH GRADE

Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 1: Geometric Figures and Their Properties – The student recognizes geometric shapes and investigates their properties including the use of concrete objects in a variety of situations.

Fourth Grade Knowledge Base Indicators	Fourth Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. recognizes and investigates properties of plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, octagons, hexagons, pentagons) using concrete objects, drawings, and appropriate technology (2.4.K1f). 2. recognizes, draws, and describes plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, octagons, hexagons, pentagons) (2.4.K1f). 3. describes the solids (cubes, rectangular prisms, cylinders, cones, spheres, triangular prisms) using the terms faces, edges, and vertices (corners) (2.4.K1f). 4. recognizes and describes the square, triangle, rhombus, hexagon, parallelogram, and trapezoid from a pattern block set (2.4.K1f). 5. recognizes (2.4.k1f): <ol style="list-style-type: none"> a. squares, rectangles, rhombi, parallelograms, trapezoids as special quadrilaterals; b. similar and congruent figures; c. points, lines (intersecting, parallel, perpendicular), line segments, and rays. 6. determines if geometric shapes and real-world objects contain line(s) of symmetry and draws the line(s) of symmetry if the line(s) exist(s) (2.4.K1f). 	<p>The student...</p> <ol style="list-style-type: none"> 1. solves real-world problems by applying the properties of (2.4.A1f): <ol style="list-style-type: none"> a. plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, parallelograms, hexagons) and lines of symmetry, e.g., print your name or the school’s name in all capital letters. Identify the lines of symmetry in each letter. b. solids (cubes, rectangular prisms, cylinders, cones, spheres), e.g., you want to design something to store school supplies. Which of the solids could you use for storage? Why did you select that solid? 2. ▲ ■ identifies the plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, octagons, hexagons, pentagons, trapezoids) used to form a composite figure (2.4.A1f).

Teacher Notes: Geometry is the study of shapes, their properties, and their relationships to other shapes. Symbols and numbers are used to describe their properties and their relationships to other shapes. The fundamental concepts in geometry are point (no dimension), line (one-dimensional), plane (two-dimensional), and space (three-dimensional). Plane figures are referred to as two-dimensional and solids are referred to as three-dimensional.

From the Mathematics Dictionary and Handbook (Nichols Schwartz Publishing, 1999), **property** as a mathematical term means a characteristic (an attribute) of a number, geometric shape, mathematical operation, equation, or inequality. To give an example:

- Property of a number: 8 is divisible by 2.
- Property of a geometric shape: Each of the four sides of a square is of the same length.
- Property of an operation: Addition is commutative. For all numbers x and y , $x + y = y + x$.
- Property of an equation: For all numbers a , b , and c , if $a = b$, then $a + c = b + c$.
- Property of an inequality: For all numbers a , b , and c , if $a > b$, then $a - c > b - c$.

Pattern blocks are a collection of six geometric shapes in six colors. Each set contains 250 pieces – 50 green triangles, 25 orange squares, 50 blue rhombi, 50 tan rhombi, 50 red trapezoids, and 25 yellow hexagons. The blue rhombus and the tan rhombus also are parallelograms, and the orange square is a parallelogram. The blocks are designed so that their sides are all the same length with the exception that the trapezoid has one side twice as long. This feature allows the blocks to be nested together and encourages the exploration of relationships among the shapes. Activities with pattern blocks help students explore patterns, functions, fractions, congruence, similarity, symmetry, perimeter, area, and graphing. A pattern block template can be found in the Appendix.

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Standard 3: Geometry

FOURTH GRADE

Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 2: Measurement and Estimation – The student estimates and measures using standard and nonstandard units of measure including the use of concrete objects in a variety of situations.

Fourth Grade Knowledge Base Indicators	Fourth Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. uses whole number approximations (estimations) for length, width, weight, volume, temperature, time, perimeter, and area using standard and nonstandard units of measure (2.4.K1a) (\$). 2. ▲ selects, explains the selection of, and uses measurement tools, units of measure, and degree of accuracy appropriate for a given situation to measure (2.4.K1a) (\$): <ol style="list-style-type: none"> a. length, width, and height to the nearest fourth of an inch or to the nearest centimeter; b. volume to the nearest cup, pint, quart, or gallon; to the nearest liter; or to the nearest whole unit of a nonstandard unit; c. weight to the nearest ounce or pound or to the nearest whole unit of a nonstandard unit of measure; d. temperature to the nearest degree; e. time including elapsed time. 3. states: <ol style="list-style-type: none"> a. the number of weeks in a year; b. the number of ounces in a pound; c. the number of milliliters in a liter, grams in a kilogram, and meters in a kilometer; d. the number of items in a dozen. 4. converts (2.4.K1a): <ol style="list-style-type: none"> a. within the customary system: inches and feet, feet and yards, inches and yards, cups and pints, pints and quarts, quarts and gallons; b. within the metric system: centimeters and meters. 5. finds(2.4.K1f): <ol style="list-style-type: none"> a. the perimeter of two-dimensional figures given the measures of all the sides. b. the area of squares and rectangles using concrete objects. 	<p>The student...</p> <ol style="list-style-type: none"> 1. solves real-world problems by applying appropriate measurements: <ol style="list-style-type: none"> a. length to the nearest fourth of an inch (2.4.A1a), e.g., how much longer is the math textbook than the science textbook? b. length to the nearest centimeter (2.4.A1a), e.g., a new pencil is about how many centimeters long? c. temperature to the nearest degree (2.4.A1a), e.g., what would the temperature outside be if it was a good day for sledding? d. weight to the nearest whole unit (pounds, grams, nonstandard unit) (2.4.A1a), e.g., Brendan went to the store and bought 2 packages of hamburger for a meatloaf. One of the hamburger packages weighed 1 lb. and 8 ozs. The other packages weighed 1 lb. and 7 ozs. What is the combined weight (to the nearest pound) of the two packages of hamburger? e. time including elapsed time (2.4.A1a), e.g., Joy went to the mall at 10:00 a.m. She shopped until 4:15 p.m. How long did she shop at the mall? f. months in a year (2.4.A1a), e.g., if it takes 208 weeks to get a college degree, and Susan has completed one year, how many more weeks does she have to complete to get her degree? g. minutes in an hour (2.4.A1a), e.g., Bob has spent 240 minutes working on a project for Science. How many hours has he worked on the project? h. perimeter of squares, rectangles, and triangles (2.4.A1f), e.g., a triangle has 3 equal sides of 32 inches. What is the perimeter of the triangle? 2. ▲ estimates to check whether or not measurements and calculations for length, width, weight, volume, temperature, time, and perimeter in real-world problems are reasonable (2.4.A1a) (\$), e.g., which is the most reasonable weight for your scissors – 2 ounces, 2 pounds, 20 ounces, or 20 pounds? A teacher measures one side of a square desktop at 2 feet. Which of the following perimeters is reasonable for the desktop – 2 feet, 4 square feet, 6 square feet, or

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■ – Assessed Indicator on the Optional Constructed Response Assessment

N – Noncalculator

(**\$**) – Financial Literacy

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	<p>8 feet?</p> <p>3. adjusts original measurement or estimation for length, width, weight, volume, temperature, time, and perimeter in real-world problems based on additional information (a frame of reference) (2.4.A1a) (\$), e.g., your class has a large jar and a small jar. You estimate it will take 5 small jars of liquid to fill the large jar. After you pour the contents of 2 small jars in, the large jar is more than half full. Should you need to adjust your estimate?</p>
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Teacher Notes: The term *geometry* comes from two Greek words meaning “earth measure.” **Measurement** provides the tools required to apply geometric concepts in the real-world. **Estimation in measurement** is defined as making guesses as to the exact measurement of an object without using any type of measurement tool. Estimation helps students develop a relationship between the different sizes of units of measure. It helps students develop basic properties of measurement and it gives students a tool to determine whether a given measurement is reasonable.

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Standard 3: Geometry

FOURTH GRADE

Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 3: Transformational Geometry – The student recognizes and performs one transformation on simple shapes or concrete objects in a variety of situations.

Fourth Grade Knowledge Base Indicators	Fourth Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> describes a transformation using cardinal points or positional directions (2.4.K1a), e.g., go north three blocks and then west four blocks or move the triangle three units to the right and two units up. ▲ ■ recognizes, performs, and describes one transformation (reflection/flip, rotation/turn, translation/slide) on a two-dimensional figure or concrete object (2.4.K1a). recognizes three-dimensional figures (rectangular prisms, cylinders) and concrete objects from various perspectives (top, bottom, sides, corners) (2.4.K1f). 	<p>The student...</p> <ol style="list-style-type: none"> recognizes real-world transformations (reflection/flip, rotation/turn, translation/slide) (2.4.A1a). gives and uses cardinal points or positional directions to move from one location to another on a map or grid (2.4.A1a). describes the properties of geometric shapes or concrete objects that stay the same and the properties that change when a transformation is performed (2.4.A1f).
<p>Teacher Notes: Transformational geometry is another way to investigate geometric figures by moving every point in a plane figure to a new location. To help students form images of shapes through different transformations, students can use concrete objects, figures drawn on graph paper, mirrors or other reflective surfaces, or appropriate technology.</p> <p>Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, <i>process models</i> are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.</p> <p>The National Standards in Personal Finance identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.</p>	

- ▲ – Assessed Indicator
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- N – Noncalculator
- (\$)

Standard 3: Geometry

FOURTH GRADE

Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 4: Geometry From An Algebraic Perspective – The student relates geometric concepts to a number line and the first quadrant of a coordinate plane in a variety of situations.

Fourth Grade Knowledge Base Indicators	Fourth Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> uses a number line (horizontal/vertical) to model whole number multiplication facts from 1 x 1 through 12 x 12 and corresponding division facts (2.4.K1a). uses points in the first quadrant of a coordinate plane (coordinate grid) to identify locations (2.4.K1a). ▲ ■ identifies and plots points as whole number ordered pairs in the first quadrant of a coordinate plane (coordinate grid) (2.4.K1a). organizes whole number data using a T-table and plots the ordered pairs in the first quadrant of a coordinate plane (coordinate grid) (2.4.K1a,e). 	<p>The student...</p> <ol style="list-style-type: none"> solves real-world problems that involve distance and location using coordinate planes (coordinate grids) and map grids with positive whole number and letter coordinates (2.4.A1a), e.g., identifying locations and giving and following directions to move from one location to another. solves real-world problems by plotting whole number ordered pairs in the first quadrant of a coordinate plane (coordinate grid) (2.4.A1a) (\$), e.g., given that each movie ticket cost \$5, the student graphs the number of tickets bought and the total cost of tickets to attend a movie. <div data-bbox="1234 760 1669 1193" style="text-align: center;"> </div>

Teacher Notes: A **number line** (a mathematical model) is a diagram that represents numbers with equal distances marked off as points on a line, and is an example of one-to-one correspondence (a relation). A number line can be used as a visual representation of numbers and operations. In addition, a number line used horizontally and vertically is a precursor to the coordinate plane; and the distance between two numbers on a number line is a precursor to absolute value.

A **coordinate plane** (coordinate grid) consists of a horizontal number line called the x-axis and a vertical number line called the y-axis. These two lines intersect at a point called the origin. The x-axis and the y-axis divide the plane into four sections called quadrants. Any point on the coordinate plane can be named with two numbers called coordinates. The first number is the x-coordinate. The second number is the y-coordinate. Since the pair is always named in order (first x, then y), it is called an ordered pair.

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Standard 4: Data

FOURTH GRADE

Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.

Benchmark 1: Probability – The student applies the concepts of probability to draw conclusions and to make predictions and decisions including the use of concrete objects in a variety of situations.

Fourth Grade Knowledge Base Indicators	Fourth Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. recognizes that the probability of an impossible event is zero and that the probability of a certain event is one (2.4.K1g) (\$). 2. lists all possible outcomes of a simple event in an experiment or simulation including the use of concrete objects (2.4.K1g-h). 3. recognizes and states the probability of a simple event in an experiment or simulation (2.4.K1g), e.g., when a coin is flipped, the probability of landing heads up is $\frac{1}{2}$ and the probability of landing tails up is $\frac{1}{2}$. This can be read as one out of two or one half. 	<p>The student...</p> <ol style="list-style-type: none"> 1. makes predictions about a simple event in an experiment or simulation; conducts an experiment or simulation including the use of concrete objects; records the results in a chart, table, or graph; and uses the results to draw conclusions about the event (2.4.A1g-h). 2. uses the results from a completed experiment or simulation of a simple event to make predictions in a variety of real-world problems (2.4.A1g-h), e.g., the manufacturer of Crunchy Flakes puts a prize in 20 out of every 100 boxes. What is the probability that a shopper will find a prize in a box of Crunchy Flakes, if they purchase 10 boxes? 3. compares what should happen (theoretical probability/expected results) with what did happen (empirical probability/experimental results) in an experiment or simulation with a simple event (2.4.A1g).

Teacher Notes: Ideas from **probability** reinforce concepts in the other Standards, especially Number and Computation and Geometry. Students need to develop an intuitive concept of chance – whether or not something is unlikely or likely to happen. Probability experiences should be addressed through the use of concrete objects, coins, and geometric models (spinners, number cubes, or dartboards). Probabilities are ratios, expressed as fractions, decimals, or percents, determined by considering results or outcomes of experiments. Some examples of uses of probability in every day life include: There is a 50% chance of rain today. What is the probability that the team will win every game?

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Standard 4: Data

FOURTH GRADE

Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.

Benchmark 2: Statistics – The student collects, organizes, displays, explains, and interprets numerical (whole numbers) and non-numerical data sets including the use of concrete objects in a variety of situations.

Fourth Grade Knowledge Base Indicators	Fourth Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. ▲ ■ organizes, displays, and reads numerical (quantitative) and non-numerical (qualitative) data in a clear, organized, and accurate manner including a title, labels, categories, and whole number intervals using these data displays (2.4.K1h) (\$): <ol style="list-style-type: none"> a. graphs using concrete objects, (for testing, does not have to use concrete objects in items); b. pictographs with a symbol or picture representing one, two, five, ten, twenty-five, or one-hundred including partial symbols when the symbol represents an even amount; c. frequency tables (tally marks); d. horizontal and vertical bar graphs; e. Venn diagrams or other pictorial displays, e.g., glyphs; f. line plots; g. charts and tables; h. line graphs; i. circle graphs. 2. collects data using different techniques (observations, polls, surveys, interviews, or random sampling) and explains the results (2.4.K1h) (\$). 3. identifies, explains, and calculates or finds these statistical measures of a data set with less than ten whole number data points using whole numbers from 0 through 1,000 (2.4.K1a) (\$): <ol style="list-style-type: none"> a. minimum and maximum values, b. range, c. mode, d. median when data set has an odd number of data points, e. mean when data set has a whole number mean. 	<p>The student...</p> <ol style="list-style-type: none"> 1. interprets and uses data to make reasonable inferences and predictions, answer questions, and make decisions from these data displays (2.4.A1h) (\$): <ol style="list-style-type: none"> a. graphs using concrete objects; b. pictographs with a symbol or picture representing one, two, five, ten, twenty-five, or one-hundred including partial symbols when the symbol represents an even amount; c. frequency tables (tally marks); d. horizontal and vertical bar graphs; e. Venn diagrams or other pictorial displays; f. line plots; g. charts and tables; h. line graphs. 2. ▲ uses these statistical measures of a data set using whole numbers from 0 through 1,000 with less than ten whole number data points to make reasonable inferences and predictions, answer questions, and make decisions (2.4.A1a) (\$): <ol style="list-style-type: none"> a. minimum and maximum values, b. range, c. mode, d. median when the data set has an odd number of data points, e. mean when the data set has a whole number mean. 3. recognizes that the same data set can be displayed in various formats including the use of concrete objects (2.4.A1h) (\$). 4. recognizes and explains the effects of scale and interval changes on graphs of whole number data sets (2.4.A1h).

Teacher Notes: Graphs (data displays) are pictorial representations of mathematical relationships, are used to tell a story, and are an important part of statistics. When a graph is made, the axes and the scale (numbers running along a side of the graph) are chosen for a reason. The difference between numbers from one grid line to another is the **interval**. The interval will depend on the lowest and highest values in the data set. Emphasizing the importance of using equal-sized pictures or intervals is critical to ensuring that the data display is accurate.

Graphs take many forms:

- bar graphs and pictographs compare discrete data,
- frequency tables show how many times a certain piece of data occurs,
- circle graphs (pie charts) model parts of a whole,
- line graphs show change over time,
- Venn diagrams show relationships between sets of objects, and
- line plots show frequency of data on a number line.

An important aspect of data is its *center*. The measures of central tendency (averages) of a data set are mean, median, and mode. Conceptual understanding of mean, median, and mode is developed through the use of concrete objects that represent the data values.

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