

Standard 1: Number and Computation

SEVENTH GRADE

Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 1: Number Sense – The student demonstrates number sense for rational numbers, the irrational number pi, and simple algebraic expressions in one variable in a variety of situations.

Seventh Grade Knowledge Base Indicators	Seventh Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. knows, explains, and uses equivalent representations for rational numbers and simple algebraic expressions including integers, fractions, decimals, percents, and ratios; integer bases with whole number exponents; positive rational numbers written in scientific notation with positive integer exponents; time; and money (2.4.K1a-c) (\$), e.g., 253,000 is equivalent to 2.53×10^5 or $x + 5x$ is equivalent to $6x$. 2. compares and orders rational numbers and the irrational number pi (2.4.K1a) (\$). 3. explains the relative magnitude between rational numbers and between rational numbers and the irrational number pi (2.4.K1a). 4. knows and explains what happens to the product or quotient when (2.4.K1a): <ol style="list-style-type: none"> a. a whole number is multiplied or divided by a rational number greater than zero and less than one, b. a whole number is multiplied or divided by a rational number greater than one, c. a rational number (excluding zero) is multiplied or divided by zero. 5. explains and determines the absolute value of rational numbers (2.4.K1a). 	<p>The student...</p> <ol style="list-style-type: none"> 1. generates and/or solves real-world problems using (2.4.A1a) (\$): <ol style="list-style-type: none"> a. ▲ equivalent representations of rational numbers and simple algebraic expressions, e.g., you are in the mountains. Wilson Mountain has an altitude of 5.28×10^3 feet. Rush Mountain is 4,300 feet tall. How much higher is Wilson Mountain than Rush Mountain? b. fraction and decimal approximations of the irrational number pi, e.g., Mary measured the distance around her 48-inch diameter circular table to be 150 inches. Using this information, approximate pi as a fraction and as a decimal. 2. determines whether or not solutions to real-world problems using rational numbers, the irrational number pi, and simple algebraic expressions are reasonable (2.4.A1a) (\$), e.g., a sweater that cost \$15 is marked 1/3 off. The cashier charged \$12. Is this reasonable?

Teacher Notes: Number sense refers to one's ability to reason with numbers and to work with numbers in a flexible way. The ability to compute mentally, to estimate based on understanding of number relationships and magnitudes, and to judge reasonableness of answers are all involved in number sense.

The student with number sense will look at a problem holistically before confronting the details of the problem. The student will look for relationships among the numbers and operations and will consider the context in which the question was posed. Students with number sense will choose or even invent a method that takes advantage of their own understanding of the relationships between numbers and between numbers and operations, and they will seek the most efficient representation for the given task. Number sense can also be recognized in the students' use of benchmarks to judge number magnitude (e.g., $\frac{2}{5}$ of 49 is less than half of 49), to recognize unreasonable results for calculations, and to employ non-standard algorithms for mental computation and estimation. (Developing Number Sense: Addenda Series, Grades 5-8, NCTM, 1991)

At this grade level, rational numbers include positive and negative numbers and very large numbers [ten million (10^7)] and very small numbers [hundred-thousandth (10^{-5})]. **Relative magnitude** refers to the size relationship one number has with another – is it much larger, much smaller, close, or about the same? For example, using the numbers 219, 264, and 457, answer questions such as

- Which two are closest? Why?
- Which is closest to 300? To 250?
- About how far apart are 219 and 500? 5,000?
- If these are 'big numbers,' what are small numbers? Numbers about the same? Numbers that make these seem small?

(Elementary and Middle School Mathematics, John A. Van de Walle, Addison Wesley Longman, Inc., 1998)

Mathematical models such as concrete objects, pictures, diagrams, Venn diagrams, number lines, hundred charts, base ten blocks, or factor trees are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.

Standard 1: Number and Computation

SEVENTH GRADE

Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 2: Number Systems and Their Properties – The student demonstrates an understanding of the rational number system and the irrational number pi; recognizes, uses, and describes their properties; and extends these properties to algebraic expressions in one variable.

Seventh Grade Knowledge Base Indicators	Seventh Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. knows and explains the relationships between natural (counting) numbers, whole numbers, integers, and rational numbers using mathematical models (2.4.K1a,k), e.g., number lines or Venn diagrams. 2. classifies a given rational number as a member of various subsets of the rational number system (2.4.K1a,k), e.g., $\sqrt{7}$ is a rational number and an integer. 3. names, uses, and describes these properties with the rational number system and demonstrates their meaning including the use of concrete objects (2.4.K1a) (\$): <ol style="list-style-type: none"> a. commutative properties of addition and multiplication (changing the order of the numbers does not change the solution); b. associative properties of addition and multiplication (changing the grouping of the numbers does not change the solution); c. distributive property [distributing multiplication or division over addition or subtraction, e.g., $2(4 - 1) = 2(4) - 2(1) = 8 - 2 = 6$]; d. substitution property (one name of a number can be substituted for another name of the same number), e.g., if $a = 2$, then $3a = 3 \times 2 = 6$. 4. uses and describes these properties with the rational number system and demonstrates their meaning including the use of concrete objects (2.4.K1a) (\$): <ol style="list-style-type: none"> a. identity properties for addition and multiplication (additive identity – zero added to any number is equal to that number; multiplicative identity – one multiplied by any number is equal to that number); b. symmetric property of equality (if $7 + 2x = 9$ then $9 = 7 + 2x$); 	<p>The student...</p> <ol style="list-style-type: none"> 1. generates and/or solves real-world problems with rational numbers and the irrational number pi using the concepts of these properties to explain reasoning (2.4.K1a) (\$): <ol style="list-style-type: none"> a. commutative and associative properties of addition and multiplication, e.g., at a delivery stop, Sylvia pulls out a flat of eggs. The flat has 5 columns and 6 rows of eggs. Express how to find the number of eggs in 2 ways. b. distributive property, e.g., trim is used around the outside edges of a bulletin board with dimensions 3 ft by 5 ft. Explain two different methods of solving this problem. c. substitution property, e.g., $V = IR$ [Ohm's Law: voltage (V) = current (I) x resistance (R)] If the current is 5 amps ($I = 5$) and the resistance is 4 ohms ($R = 4$), what is the voltage? d. symmetric property of equality, e.g., Sam took a \$15 check to the bank and received a \$10 bill and a \$5 bill. Later Sam took a \$10 bill and a \$5 bill to the bank and received a check for \$15. $\\$15 = \\$10 + \\$5$ is the same as $\\$10 + \\$5 = \\$15$. e. additive and multiplicative identities, e.g., Bob and Sue each read the same number of books. During the week, they each read 5 more books. Compare the number of books each read. Let b=the number of books Bob read and s=the number of books Sue read, then $b+5=s+5$ f. zero property of multiplication, e.g., Jenny was thinking of two numbers. Jenny said that the product of the two numbers was 0. What could you deduct from this statement? Explain your reasoning.

<p>c. zero property of multiplication (any number multiplied by zero is zero);</p> <p>d. addition and multiplication properties of equality (adding/multiplying the same number to each side of an equation results in an equivalent equation);</p> <p>e. additive and multiplicative inverse properties. (Every number has a value known as its additive inverse and when the original number is added to that additive inverse, the answer is zero, e.g., $+5 + ^-5 = 0$. Every number except 0 has a value known as its multiplicative inverse and when the original number multiplied by its inverse, the answer will be 1, e.g., $8 \times 1/8 = 1$.)</p> <p>5. recognizes that the irrational number pi can be represented by approximate rational values, e.g., $22/7$ or 3.14.</p>	<p>g. addition and multiplication properties of equality, e.g., the total price (P) of a car, including tax (T), is \$14, 685. 33. If the tax is \$785.42, what is the sale price of the car (S)?</p> <p>h. additive and multiplicative inverse properties, e.g., if 5 candy bars cost \$1.00, what does one candy bar cost? Explain your reasoning.</p> <p>2. analyzes and evaluates the advantages and disadvantages of using integers, whole numbers, fractions (including mixed numbers), decimals, or the irrational number pi and its rational approximations in solving a given real-world problem (2.4.K1a, e.g., in the store everything is 25% off. When calculating the discount, which representation of 25% would you use and why?)</p>
<p>Teacher Notes: From the Mathematics Dictionary and Handbook (Nichols Schwartz Publishing, 1999), property as a mathematical term means a characteristic (an attribute) of a number, geometric shape, mathematical operation, equation, or inequality. To give an example:</p> <ul style="list-style-type: none"> • Property of a number: 8 is divisible by 2. • Property of a geometric shape: Each of the four sides of a square is of the same length. • Property of an operation: Addition is commutative. For all numbers x and y, $x + y = y + x$. • Property of an equation: For all numbers a, b, and c, if $a = b$, then $a + c = b + c$. • Property of an inequality: For all numbers a, b, and c, if $a > b$, then $a - c > b - c$. <p>Mathematical models such as concrete objects, pictures, diagrams, Venn diagrams, number lines, hundred charts, base ten blocks, or factor trees are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, <i>process models</i> are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.</p> <p>The National Standards in Personal Finance identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$) . The National Standards in Personal Finance are included in the Appendix.</p>	

Standard 1: Number and Computation

SEVENTH GRADE

Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 3: Estimation – The student uses computational estimation with rational numbers and the irrational number pi in a variety of situations.

Seventh Grade Knowledge Base Indicators	Seventh Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> estimates quantities with combinations of rational numbers and/or the irrational number pi using various computational methods including mental math, paper and pencil, concrete objects, and/or appropriate technology (2.4.K1a) (\$). N uses various estimation strategies and explains how they were used to estimate rational number quantities and the irrational number pi (2.4.K1a) (\$). recognizes and explains the difference between an exact and approximate answer (2.4.K1a). determines the appropriateness of an estimation strategy used and whether the estimate is greater than (overestimate) or less than (underestimate) the exact answer and its potential impact on the result (2.4.K1a). knows and explains why the fraction (22/7) or decimal (3.14) representation of the irrational number pi is an approximate value (2.4.K1c). 	<p>The student...</p> <ol style="list-style-type: none"> adjusts original rational number estimate of a real-world problem based on additional information (a frame of reference) (2.4.A1a) (\$), e.g., estimate the weight of a bookshelf of books. Then weigh one book and adjust your estimate. estimates to check whether or not the result of a real-world problem using rational numbers, the irrational number pi, and/or simple algebraic expressions is reasonable and makes predictions based on the information (2.4.A1a), e.g., a goat is staked out in a pasture with a rope that is 7 feet long. The goat needs 200 square feet of grass to graze. Does the goat have enough pasture? If not, how long should the rope be? determines a reasonable range for the estimation of a quantity given a real-world problem and explains the reasonableness of the range (2.4.A1a), e.g., how long will it take your teacher to walk two miles? The range could be 25-35 minutes. determines if a real-world problem calls for an exact or approximate answer and performs the appropriate computation using various computational methods including mental math, paper and pencil, concrete objects, and/or appropriate technology (2.4.A1a) (\$), e.g., Kathy buys items at the grocery store priced at \$32.56, \$12.83, \$6.99, and 5 for \$12.49 each. She has \$120 with her to pay for the groceries. To decide if she can pay for her items, does she need an exact or an approximate answer?

Teacher Notes: Estimate, as a verb, means to make an educated guess based on information in a problem or to give an answer close to the exact number. Estimation is used when an exact answer is not needed, as in many real-life situations for which “ballpark” computations are acceptable. Good number sense enables one to estimate a quantity, estimate a measure, or estimate an answer.

Estimation serves as an important companion to computation. It provides a tool for judging the reasonableness of computational methods including mental math, paper and pencil, concrete objects, and appropriate technology. However, being able to compute does not automatically lead to an ability to estimate or judge reasonableness of answers. Frequent modeling by the teacher helps students develop a range of estimation strategies. Students should be encouraged to frequently explain their thinking as they estimate. As with exact computation, sharing estimation strategies allows students access to others’ thinking and provides opportunities for class discussion. Identifying the estimation strategy by name is not critical; however, explaining the thinking behind the strategy to make a valid estimation is important. (Principles and Standards for School Mathematics, NCTM, 2000)

Mental math and **estimation** are distinct but related mathematical skills. Proficiency in mental math contributes to increased skill in estimation. In order for students to become more familiar with estimation, teachers should introduce estimation with examples where rounded or estimated numbers are used. Emphasis should be placed on real-world examples where only estimation is required, e.g., About how many hours do you sleep a night? Using the language of estimation is important, so students begin to realize that a variety of estimates (answers) are possible. In addition, when students are taught specific estimation strategies, they develop mental math and estimation skills easier. Estimation strategies include front-end with adjustment, compatible “nice” numbers, clustering, special numbers, or truncation.

Mathematical models such as concrete objects, pictures, diagrams, Venn diagrams, number lines, hundred charts, base ten blocks, or factor trees are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.

Standard 1: Number and Computation

SEVENTH GRADE

Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

Benchmark 4: Computation – The student models, performs, and explains computation with rational numbers, the irrational number pi, and first-degree algebraic expressions in one variable in a variety of situations.

Seventh Grade Knowledge Base Indicators	Seventh Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. computes with efficiency and accuracy using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.K1a-c) (\$). 2. performs and explains these computational procedures (2.4.K1a): <ol style="list-style-type: none"> a. ▲N adds and subtracts decimals from ten millions place through hundred thousandths place; b. ▲N multiplies and divides a four-digit number by a two-digit number using numbers from thousands place through thousandths place; c. ▲N multiplies and divides using numbers from thousands place through thousandths place by 10; 100; 1,000; .1; .01; .001; or single-digit multiples of each, e.g., $54.2 \div .002$ or 54.3×300; d. ▲N adds, subtracts, multiplies, and divides fractions and expresses answers in simplest form; e. N adds, subtracts, multiplies, and divides integers; f. N uses order of operations (evaluates within grouping symbols, evaluates powers to the second or third power, multiplies or divides in order from left to right, then adds or subtracts in order from left to right) using whole numbers; g. simplifies positive rational numbers raised to positive whole number powers; h. combines like terms of a first degree algebraic expression. 3. recognizes, describes, and uses different ways to express computational procedures, e.g., $5 - 2 = 5 + (-2)$ or $49 \times 23 = (40 \times 23) + (9 \times 23)$ or $49 \times 23 = (49 \times 20) + (49 \times 3)$ or $49 \times 23 = (50 \times 23) - 23$. 4. finds prime factors, greatest common factor, multiples, and the least common multiple (2.4.K1d). 	<p>The student...</p> <ol style="list-style-type: none"> 1. generates and/or solves one- and two-step real-world problems using these computational procedures and mathematical concepts (2.4.A1a) (\$): <ol style="list-style-type: none"> a. ■ addition, subtraction, multiplication, and division of rational numbers with a special emphasis on fractions and expressing answers in simplest form, e.g., at the candy store, you buy $\frac{3}{4}$ of a pound of peppermints and $\frac{1}{2}$ of a pound of licorice. The cost per pound for each kind of candy is \$3.00. What is the total cost of the candy purchased? b. addition, subtraction, multiplication, and division of rational numbers with a special emphasis on integers, e.g., the high temperatures for the week were: -4°, 10°, -1°, 0°, 7°, 3°, and -5°. What is the mean temperature for the week? c. first degree algebraic expressions in one variable, e.g., Jenny rents 3 videos plus \$20 of other merchandise. Barb rents 5 videos plus \$15 of other merchandise. Represent the total purchases of Jenny and Barb using V as the price of a video rental. d. percentages of rational numbers, e.g., if the sales tax is 5.5%, what is the sales tax on an item that costs \$36? e. approximation of the irrational number pi, e.g., what is the approximate diameter of a 400-meter circular track?

5. ▲ finds percentages of rational numbers (2.4.K1a,c) (\$), e.g., $12.5\% \times \$40.25 = n$ or 150% of 90 is what number? (For the purpose of assessment, percents will not be between 0 and 1.)

Teacher Notes: Efficiency and accuracy means that students are able to compute single-digit numbers with fluency. Students increase their understanding and skill in addition, subtraction, multiplication, and division by understanding the relationships between addition and subtraction, addition and multiplication, multiplication and division, and subtraction and division. Students learn basic number combinations and develop strategies for computing that makes sense to them. Through class discussions, students can compare the ease of use and ease of explanation of various strategies. In some cases, their strategies for computing will be close to conventional algorithms; in other cases, they will be quite different. Many times, students' invented approaches are based on a sound understanding of numbers and operations, and these invented approaches often can be used with efficiency and accuracy. (Principles and Standards for School Mathematics, NCTM, 2000)

The definition of computation is finding the standard representation for a number. For example, $6 + 6$, 4×3 , $17 - 5$, and $24 \div 2$ are all representations for the standard representation of 12. **Mental math** is mentally finding the standard representation for a number – calculating in your head instead of calculating using paper and pencil or technology. One of the main reasons for teaching mental math is to help students determine if a computed/calculated answer is reasonable; in other words, using mental math to estimate to see if the answer makes sense. Students develop mental math skills easier when they are taught specific strategies. Mental math strategies include counting on, doubling, repeated doubling, halving, making tens, multiplying by powers of ten, dividing with tens, thinking money, and using compatible “nice” numbers.

Mathematical models such as concrete objects, pictures, diagrams, Venn diagrams, number lines, hundred charts, base ten blocks, or factor trees are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

Technology is changing mathematics and its uses. The use of technology including calculators and computers is an important part of growing up in a complex society. It is not only necessary to estimate appropriate answers accurately when required, but also it is also important to have a good understanding of the underlying concepts in order to know when to apply the appropriate procedure. Technology does not replace the need to learn basic facts, to compute mentally, or to do reasonable paper-and-pencil computation. However, dividing a 5-digit number by a 2-digit number is appropriate with the exception of dividing by 10, 100, or 1,000 and simple multiples of each.

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Standard 2: Algebra

SEVENTH GRADE

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 1: Patterns – The student recognizes, describes, extends, develops, and explains the general rule of a pattern in a variety of situations.

Seventh Grade Knowledge Base Indicators	Seventh Grade Application Indicators																														
<p>The student...</p> <ol style="list-style-type: none"> identifies, states, and continues a pattern presented in various formats including numeric (list or table), algebraic (symbolic notation), visual (picture, table, or graph), verbal (oral description), kinesthetic (action), and written using these attributes: <ol style="list-style-type: none"> ▲ counting numbers including perfect squares, cubes, and factors and multiples (number theory) (2.4.K1a); ▲ positive rational numbers including arithmetic and geometric sequences (arithmetic: sequence of numbers in which the difference of two consecutive numbers is the same, geometric: a sequence of numbers in which each succeeding term is obtained by multiplying the preceding term by the same number) (2.4.K1a), e.g., 2, 1/2, 1/8, 1/32, ...; geometric figures (2.4.K1f); measurements (2.4.K1a); things related to daily life (2.4.K1a) (\$), e.g., tide, moon cycle, or temperature. generates a pattern (2.4.K1a). extends a pattern when given a rule of one or two simultaneous changes (addition, subtraction, multiplication, division) between consecutive terms (2.4.K1a), e.g., find the next three numbers in a pattern that starts with 3, where you double and add 1 to get the next number; the next three numbers are 7, 15, and 31. ▲ ■ states the rule to find the n^{th} term of a pattern with one operational change (addition or subtraction) between consecutive terms (2.4.K1a), e.g., given 3, 5, 7, and 9; the n^{th} term is $2n + 1$. (This is the explicit rule for the pattern.) 	<p>The student...</p> <ol style="list-style-type: none"> generalizes a pattern by giving the n^{th} term using symbolic notation (2.4.A1a,f), e.g., given the following, the n^{th} term is $2n$. <table style="margin-left: 40px; border-collapse: collapse;"> <tr> <td style="padding-right: 20px;">1 person has 2 eyes</td> <td style="border: 1px solid black; padding: 2px 10px;">X</td> <td style="border: 1px solid black; padding: 2px 10px;">Y</td> </tr> <tr> <td>2 people have 4 eyes</td> <td style="border: 1px solid black; padding: 2px 10px;">1</td> <td style="border: 1px solid black; padding: 2px 10px;">2</td> </tr> <tr> <td>3 people have 6 eyes</td> <td style="border: 1px solid black; padding: 2px 10px;">2</td> <td style="border: 1px solid black; padding: 2px 10px;">4</td> </tr> <tr> <td>4 people have 8 eyes</td> <td style="border: 1px solid black; padding: 2px 10px;">3</td> <td style="border: 1px solid black; padding: 2px 10px;">6</td> </tr> <tr> <td style="padding-left: 20px;">.</td> <td style="border: 1px solid black; padding: 2px 10px;">4</td> <td style="border: 1px solid black; padding: 2px 10px;">8</td> </tr> <tr> <td style="padding-left: 20px;">.</td> <td style="border: 1px solid black; padding: 2px 10px;">.</td> <td style="border: 1px solid black; padding: 2px 10px;">.</td> </tr> <tr> <td style="padding-left: 20px;">.</td> <td style="border: 1px solid black; padding: 2px 10px;">.</td> <td style="border: 1px solid black; padding: 2px 10px;">.</td> </tr> <tr> <td style="padding-left: 20px;">.</td> <td style="border: 1px solid black; padding: 2px 10px;">.</td> <td style="border: 1px solid black; padding: 2px 10px;">.</td> </tr> <tr> <td style="padding-left: 20px;">.</td> <td style="border: 1px solid black; padding: 2px 10px;">.</td> <td style="border: 1px solid black; padding: 2px 10px;">.</td> </tr> <tr> <td>n people have $2n$ eyes</td> <td style="border: 1px solid black; padding: 2px 10px;">n</td> <td style="border: 1px solid black; padding: 2px 10px;">$2n$</td> </tr> </table> recognizes the same general pattern presented in different representations [numeric (list or table), visual (picture, table, or graph), and written] (2.4.A1a,f,j-k) (\$). 	1 person has 2 eyes	X	Y	2 people have 4 eyes	1	2	3 people have 6 eyes	2	4	4 people have 8 eyes	3	6	.	4	8	n people have $2n$ eyes	n	$2n$
1 person has 2 eyes	X	Y																													
2 people have 4 eyes	1	2																													
3 people have 6 eyes	2	4																													
4 people have 8 eyes	3	6																													
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Teacher Notes: A fundamental component in the development of classification, number, and problem solving skills is inventing, discovering, and describing patterns. Patterns pervade all of mathematics and much of nature. All **patterns** are either repeating or growing or a variation of either or both. Translating a pattern from one medium to another to find two patterns that are alike, even though they are made with different materials, is important so students can see the relationships that are critical to repeating patterns. With growing patterns, students not only extend patterns, but also look for a generalization or an algebraic relationship that will tell them what the pattern will be at any point along the way.

Working with **patterns** is an important process in the development of mathematical thinking. Patterns can be based on geometric attributes (shapes, regions, angles); measurement attributes (color, texture, length, weight, volume, number); relational attributes (proportion, sequence, functions); and affective attributes (values, likes, dislikes, familiarity, heritage, culture). (Learning to Teach Mathematics, Randall J. Souviney, Macmillan Publishing Company, 1994)

In the pattern that begins with 3, 5, 7, and 9; the explicit rule is $2n + 1$ and the recursive rule is add 2 to the previous term. Patterns themselves are not explicit or recursive. The *RULE* for the pattern can be expressed explicitly or recursively and *MOST* patterns can be explained using either format especially *IF* that pattern reflects either an arithmetic sequence or geometric sequence.

This process (working with patterns) can be used to develop or deepen understandings of important concepts in number theory, rational numbers, measurement, geometry, probability, and functions. Working with patterns provides opportunities for students to recognize, describe, extend, develop, and explain.

Number theory is the study of the properties of the counting (natural) numbers, their relationships, ways to represent them, and patterns among them. Number theory includes the concepts of odd and even numbers, factors and multiples, primes and composites, greatest common factor and least common multiple, and sequences.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.

7-10
January 31, 2004

▲ – Assessed Indicator on the Objective Assessment

■ – Assessed Indicator on the Optional Constructed Response Assessment

N – Noncalculator

(\$) – Financial Literacy

THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.

Standard 2: Algebra

SEVENTH GRADE

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 2: Variable, Equations, and Inequalities – The student uses variables, symbols, rational numbers, and simple algebraic expressions in one variable to solve linear equations and inequalities in a variety of situations.

Seventh Grade Knowledge Base Indicators	Seventh Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> knows and explains that a variable can represent a single quantity that changes (2.4.K1a), e.g., daily temperature. knows, explains, and uses equivalent representations for the same simple algebraic expressions (2.4.K1a), e.g., $x + y + 5x$ is the same as $6x + y$. shows and explains how changes in one variable affects other variables (2.4.A1a), e.g., changes in diameter affects circumference. explains the difference between an equation and an expression. solves (2.4.K1a,e) (\$): <ol style="list-style-type: none"> one-step linear equations in one variable with positive rational coefficients and solutions, e.g., $7x = 28$ or $x + \frac{3}{4} = 7$ or $\frac{x}{3} = 5$; two-step linear equations in one variable with counting number coefficients and constants and positive rational solutions; one-step linear inequalities with counting numbers and one variable, e.g., $3x > 12$. explains and uses the equality and inequality symbols ($=$, \neq, $<$, \leq, $>$, \geq) and corresponding meanings (is equal to, is not equal to, is less than, is less than or equal to, is greater than, is greater than or equal to) to represent mathematical relationships with rational numbers (2.4.K1a) (\$). ▲ knows the mathematical relationship between ratios, proportions, and percents and how to solve for a missing term in a proportion with positive rational number solutions and monomials (2.4.K1a,c) (\$), e.g., $\frac{5}{6} = \frac{2}{x}$. ▲ evaluates simple algebraic expressions using positive rational numbers (2.4.K1c) (\$), e.g., if $x = 3/2$, $y = 2$, then $5xy + 2 = 5(3/2)(2) + 2 = 17$. 	<p>The student...</p> <ol style="list-style-type: none"> ▲ represents real-world problems using variables and symbols to write linear expressions, one- or two-step equations (2.4.A1e) (\$), e.g., John has three times as much money as his sister. If M is the amount of money his sister has, what is the equality that represents the amount of money that John has? To represent the problem situation, $J = 3M$ could be written. solves real-world problems with one- or two-step linear equations in one variable with whole number coefficients and constants and positive rational solutions intuitively and analytically (2.4.A1e) (\$), e.g., Kim has read 5 more than twice the number of pages as Hank. Kim has read 15 pages. How many pages has Hank read? To solve analytically, write $2h + 5 = 15$. Then find the answer. generates real-world problems that represent one- or two-step linear equations (2.4.A1e) (\$), e.g., given the equation $x + 10 = 30$, the problem could be: Two items cost \$30.00. If one item costs \$10.00, what is the cost of the other item? explains the mathematical reasoning that was used to solve a real-world problem using a one- or two-step linear equation (2.4.A1e) (\$), e.g., Kim has read 5 more than twice the number of pages as Hank. Kim has read 15 pages. How many pages has Hank read? To solve, write $2h + 5 = 15$. Let h=number of pages Hank read. Then to find the answer subtract 5 from both sides of the equation. $2h = 10$, then divide both sides of the equation by 2, so $h = 5$.

7-11
January 31, 2004

▲ – Assessed Indicator on the Objective Assessment

■ – Assessed Indicator on the Optional Constructed Response Assessment

N – Noncalculator

(\$) – Financial Literacy

THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.

Teacher Notes: Understanding the **concept of variable** is fundamental to algebra. Students use various symbols, including letters and geometric shapes to represent unknown quantities that both do and do not vary. Quantities that are not given and do not vary are often referred to as unknowns or missing elements when they appear in equations, e.g., $2 + \Delta = 4$ or $3 \cdot s = 15$ where a triangle and s are used as variables. Various symbols or letters should be used interchangeably in equations.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

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7-12
January 31, 2004

▲ – Assessed Indicator on the Objective Assessment

■ – Assessed Indicator on the Optional Constructed Response Assessment

N – Noncalculator

(\$) – Financial Literacy

THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.

Standard 2: Algebra

SEVENTH GRADE

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 3: Functions – The student recognizes, describes, and analyzes constant and linear relationships in a variety of situations.

Seventh Grade Knowledge Base Indicators	Seventh Grade Application Indicators																								
<p>The student...</p> <ol style="list-style-type: none"> recognizes constant and linear relationships using various methods including mental math, paper and pencil, concrete objects, and graphing utilities or appropriate technology (2.4.K1a,e-g) (\$). finds the values and determines the rule through two operations using a function table (input/output machine, T-table) (2.4.K1f). demonstrates mathematical relationships using ordered pairs in all four quadrants of a coordinate plane (2.4.K1g). describes and/or gives examples of mathematical relationships that remain constant (2.4.K1e-g) (\$), e.g., you will get \$10.00 to do a job, no matter how long it takes for you to do it. 	<p>The student...</p> <ol style="list-style-type: none"> represents a variety of constant and linear relationships using written or oral descriptions of the rule, tables, graphs, and when possible, symbolic notation (2.4.A13-g,k) (\$), e.g., the relationship between cars and their wheels (written) becomes a table: <table style="margin-left: 40px; border-collapse: collapse;"> <thead> <tr> <th style="border-right: 1px solid black; padding: 2px 10px;">Cars</th> <th style="padding: 2px 10px;">Wheels</th> <th style="padding: 2px 10px;"></th> </tr> </thead> <tbody> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">4</td> <td style="padding: 2px 10px;">→ (1,4)</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;">2</td> <td style="padding: 2px 10px;">8</td> <td style="padding: 2px 10px;">→ (2,8)</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;">10</td> <td style="padding: 2px 10px;">40</td> <td style="padding: 2px 10px;">→ (10,40)</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;">.</td> <td style="padding: 2px 10px;">.</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;">.</td> <td style="padding: 2px 10px;">.</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;">.</td> <td style="padding: 2px 10px;">.</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;">n</td> <td style="padding: 2px 10px;">4n</td> <td></td> </tr> </tbody> </table> <p>and then the ordered pairs of (1,4), (2,8), (10,40), and (n,4n) can be graphed.</p> interprets, describes, and analyzes the mathematical relationships of numerical, tabular, and graphical representations, including translations between the representations (2.4.A1k) (\$). 	Cars	Wheels		1	4	→ (1,4)	2	8	→ (2,8)	10	40	→ (10,40)		n	4n	
Cars	Wheels																								
1	4	→ (1,4)																							
2	8	→ (2,8)																							
10	40	→ (10,40)																							
.	.																								
.	.																								
.	.																								
n	4n																								

Teacher Notes: Functions are relationships or rules in which each member of one set is paired with one, and only one, member of another set (an ordered pair). The concept of function can be introduced using function machines. Any number put in the machine will be changed according to some rule. A record of inputs and corresponding outputs can be maintained in a two-column format. Function tables, input/output machines, and T-tables may be used interchangeably and serve the same purpose.

Function concepts should be developed from **growing patterns**. Each term in a number sequence is related to its position in the sequence – the functional relationship. The pattern – 4, 7, 10, 13, 16, 19, and so on – is an arithmetic sequence *with a difference of 3*. The pattern could be described as *add 3* meaning that 3 must be added to the previous term to find the next. This pattern is explained by using the recursive definition for a sequence. The recursive definition for a sequence is a statement or a set of statements that explains how each successive term in the sequence is obtained from the previous term(s).

In the pattern 1, 4, 9, 16, 25, ..., 225; there is *no common difference*. This sequence is not arithmetic or geometric (no common ratio between geometric terms). Neither is it a combination of the two; however, there is a pattern and the missing terms between 25 and 225 can be found. To find the term value, square the number of the term. The next missing terms would be 36, 49, 64, 81, 100, 121, and 144. This pattern is explained by using the explicit formula for a sequence. The explicit formula for a sequence defines a rule for finding each term in the number sequence related to its position in the sequence. In other words, to find the term value, square the number of the term – the 5th term is 5², the 8th term is 8², ...

Patterns themselves are not explicit or recursive. The *RULE* for the pattern can be expressed explicitly or recursively and *MOST* patterns can be explained using either format especially *IF* that pattern reflects either an arithmetic sequence or geometric sequence.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$) . The National Standards in Personal Finance are included in the Appendix.

7-14
January 31, 2004

▲ – Assessed Indicator on the Objective Assessment

■ – Assessed Indicator on the Optional Constructed Response Assessment

N – Noncalculator

(§) – Financial Literacy

THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.

Standard 2: Algebra

SEVENTH GRADE

Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

Benchmark 4: Models – The student generates and uses mathematical models to represent and justify mathematical relationships found in a variety of situations.

Seventh Grade Knowledge Base Indicators	Seventh Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. knows, explains, and uses mathematical models to represent and explain mathematical concepts, procedures, and relationships. Mathematical models include: <ol style="list-style-type: none"> a. process models (concrete objects, pictures, diagrams, number lines, hundred charts, measurement tools, multiplication arrays, division sets, or coordinate grids) to model computational procedures, algebraic relationships, and mathematical relationships and to solve equations (1.1.K1-5, 1.2.K1-4, 1.3.K1-4, 1.4.K1-2, 1.4.K5, 2.1.K1a-b, 2.1.K1e, 2.1.K2-4, 2.2.K1-3, 2.2.K5-6, 2.3.K1, 3.1.K9, 3.2.K1-3, 3.2.K9, 3.3.K1-4, 3.4.K1, 4.2.K4-6) (\$); b. place value models (place value mats, hundred charts, base ten blocks, or unifix cubes) to compare, order, and represent numerical quantities and to model computational procedures (1.1.K1, 1.4.K2) (\$); c. fraction and mixed number models (fraction strips or pattern blocks) and decimal and money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.K1, 1.3.K5, 1.4.K2, 2.2.K7-8, 4.1.K3) (\$); d. factor trees to find least common multiple and greatest common factor and to model prime factorization (1.4.K4); e. equations and inequalities to model numerical relationships (2.2.K5, 2.3.K1, 2.3.K4) (\$); f. function tables to model numerical and algebraic relationships (2.3.K1-2, 2.3.K4) (\$); g. coordinate planes to model relationships between ordered pairs and linear equations (2.3.K1, 2.3.K3-4, 3.4.K2-4) (\$); h. two- and three-dimensional geometric models (geoboards, dot paper, nets or solids) to model perimeter, area, volume, and surface area, and properties of two- and three-dimensional (2.1.K1c, 3.1.K1, 3.1.K3-8, 3.1.K10, 3.2.K1-2, 3.2.K4-8, 3.2.K10); 	<p>The student...</p> <ol style="list-style-type: none"> 1. recognizes that various mathematical models can be used to represent the same problem situation. Mathematical models include: <ol style="list-style-type: none"> a. process models (concrete objects, pictures, diagrams, flowcharts, number lines, hundred charts, measurement tools, multiplication arrays, division sets, or coordinate grids) to model computational procedures, algebraic relationships, mathematical relationships, and problem situations and to solve equations (1.1.A1, 1.2.A1-2, 1.3.A1-4, 1.4.A1, 2.1.A1-2, 3.1.A1, 3.2.A1a, 3.2.A1d, 3.2.A1f, 3.2.A2, 3.3.A1, 4.2.A4) (\$); b. place value models (place value mats, hundred charts, base ten blocks, or unifix cubes) to model problem situations (1.1.A1a, 1.1.A2a, 1.2.A12, 1.3.A1.2, 1.4.A3a-e, 2.2.A3) (\$); c. fraction and mixed number models (fraction strips or pattern blocks) and decimal and money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.A1b, 1.1.A2b, 1.2.A1-2, 1.3.A1-2) (\$); d. factor trees to find least common multiple and greatest common factor and to model prime factorization (1.4.K5) e. equations and inequalities to model numerical relationships (2.2.A1-4, 2.3.A1, 3.2.A1e). (\$) f. function tables to model numerical and algebraic relationships (2.1.A1-2, 2.3.A1) (\$); g. coordinate planes to model relationships between ordered pairs and linear equations (2.3.A1, 3.4.A1) (\$); h. two- and three-dimensional geometric models (geoboards, dot paper, nets or solids) to model perimeter, area, volume, and surface area, and properties of two- and three-dimensional models (3.1.A2-3, 3.2.A1b-c, 3.2.A1e, 3.3.A2, 3.4.A1); i. scale drawings to model large and small real-world objects (3.3.A3);

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N – Noncalculator

(\$) – Financial Literacy

THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.

<ul style="list-style-type: none"> i. geometric models (spinners, targets, or number cubes), process models (coins, pictures, or diagrams), and tree diagrams to model probability (4.1.K1, 4.1.K4) (\$); j. frequency tables, bar graphs, line graphs, circle graphs, Venn diagrams, charts, tables, single stem-and-leaf plots, scatter plots, and box-and-whisker plots to organize and display data (4.2.K1) (\$); k. Venn diagrams to sort data and show relationships (1.2.K1-2). 	<ul style="list-style-type: none"> j. geometric models (spinners, targets, or number cubes), process models (coins, pictures, or diagrams), and tree diagrams to model probability (4.1.A1) (\$); k. frequency tables, bar graphs, line graphs, circle graphs, Venn diagrams, charts, tables, single stem-and-leaf plots, scatter plots, and box-and-whisker plots to describe, interpret, and analyze data (2.1.A2, 2.3.A1-2, 4.1.A1-2, 4.2.A1-3) (\$); l. Venn diagrams to sort data and show relationships. <ol style="list-style-type: none"> 2. selects a mathematical model and justifies why some mathematical models are more accurate than other mathematical models in certain situations, e.g., recognizes that change over time is better represented through a line graph than through a table of ordered pairs. 3. uses the mathematical modeling process to make inferences about real-world situations when the mathematical model used to represent the situation is given.
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Teacher Notes: For assessment purposes, the mathematical modeling process appropriate to the indicator may be included as part of the item being assessed.

The **mathematical modeling** process involves:

- a. selecting key features and relationships within the real-world situation and representing these concepts in mathematical terms through some sort of mathematical model,
- b. performing manipulations and mathematical procedures within the mathematical model,
- c. interpreting the results of the manipulations within the mathematical model,
- d. using these results to make inferences about the original real-world situation.

The use of **mathematical models** is necessary for conceptual understanding. The ways in which mathematical ideas are represented is fundamental to how students understand and use those ideas. As students begin to use multiple representations of the same situation, they begin to develop an understanding of the advantages and disadvantages of various representations/models.

Many **mathematical models** are listed in this benchmark. The indicator lists some of the mathematical models that could be used to teach a concept. Each indicator in this benchmark is linked to other indicators in other benchmarks; those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3. In addition, the indicator in the other benchmarks identifies, in parentheses, the Models' indicator. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models).

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Standard 3: Geometry

SEVENTH GRADE

Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 1: Geometric Figures and Their Properties – The student recognizes geometric figures and compares their properties in a variety of situations.

Seventh Grade Knowledge Base Indicators	Seventh Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. recognizes and compares properties of two- and three-dimensional figures using concrete objects, constructions, drawings, appropriate terminology, and appropriate technology (2.4.K1h). 2. classifies regular and irregular polygons having through ten sides as convex or concave. 3. ▲ identifies angle and side properties of triangles and quadrilaterals (2.4.K1h): <ol style="list-style-type: none"> a. sum of the interior angles of any triangle is 180°; b. sum of the interior angles of any quadrilateral is 360°; c. parallelograms have opposite sides that are parallel and congruent; d. rectangles have angles of 90°, opposite sides are congruent; e. rhombi have all sides the same length, opposite angles are congruent; f. squares have angles of 90°, all sides congruent; g. trapezoids have one pair of opposite sides parallel and the other pair of opposite sides are not parallel. 4. identifies and describes (2.4.K1h): <ol style="list-style-type: none"> a. the altitude and base of a rectangular prism and triangular prism, b. the radius and diameter of a cylinder. 5. identifies corresponding parts of similar and congruent triangles and quadrilaterals (2.4.K1h). 6. uses symbols for right angle within a figure (\square), parallel (\parallel), perpendicular (\perp), and triangle (Δ) to describe geometric figures(2.4.K1h). 7. classifies triangles as (2.4.K1h): <ol style="list-style-type: none"> a. scalene, isosceles, or equilateral; b. right, acute, obtuse, or equiangular. 	<p>The student...</p> <ol style="list-style-type: none"> 1. solves real-world problems by applying the properties of (2.4.A1a): <ol style="list-style-type: none"> a. plane figures (regular and irregular polygons through 10 sides, circles, and semicircles) and the line(s) of symmetry; e.g., two guide wires are used to stabilize a tower. The wires with the ground form an isosceles triangle. The two base angles form 20° angles with the ground. What is the size of the vertex angle made where the wires meet on the tower? b. solids (cubes, rectangular prisms, cylinders, cones, spheres, triangular prisms) emphasizing faces, edges, vertices, and bases; e.g., lace is to be glued on all of the edges of a cube. If one edge measures 34 cm, how much lace is needed? 2. decomposes geometric figures made from (2.4.A1h): <ol style="list-style-type: none"> a. regular and irregular polygons through 10 sides, circles, and semicircles; b. nets (two-dimensional shapes that can be folded into three-dimensional figures), e.g., the cardboard net that becomes a shoebox; c. prisms, pyramids, cylinders, cones, spheres, and hemispheres. 3. composes geometric figures made from (2.4.A1h): <ol style="list-style-type: none"> a. regular and irregular polygons through 10 sides, circles, and semicircles; b. nets (two-dimensional shapes that can be folded into three-dimensional figures); c. prisms, pyramids, cylinders, cones, spheres, and hemispheres.

8. determines if a triangle can be constructed given sides of three different lengths(2.4.K1h).
9. generates a pattern for the sum of angles for 3-, 4-, 5-, ... n-sides polygons (2.4.K1a).
10. describes the relationship between the diameter and the circumference of a circle (2.4.K1h).

Teacher Notes: Geometry is the study of shapes, their properties, and their relationships to other shapes. Symbols and numbers are used to describe their properties and their relationships to other shapes. The fundamental concepts in geometry are point (no dimension), line (one-dimensional), plane (two-dimensional), and space (three-dimensional). Plane figures are referred to as two-dimensional. Solids are referred to as three-dimensional. The base, in terms of geometry, generally refers to the side on which a figure rests. Therefore, depending on the orientation of the solid, the base changes.

From the Mathematics Dictionary and Handbook (Nichols Schwartz Publishing, 1999), **property** as a mathematical term means a characteristic (an attribute) of a number, geometric shape, mathematical operation, equation, or inequality. To give an example:

- Property of a number: 8 is divisible by 2.
- Property of a geometric shape: Each of the four sides of a square is of the same length.
- Property of an operation: Addition is commutative. For all numbers x and y , $x + y = y + x$.
- Property of an equation: For all numbers a , b , and c , if $a = b$, then $a + c = b + c$.
- Property of an inequality: For all numbers a , b , and c , if $a > b$, then $a - c > b - c$.

The application of the Knowledge Indicators from the Geometry Benchmark, Geometric Figures and Their Properties are most often applied within the context of the other Geometry Benchmarks — Measurement and Estimation, Transformational Geometry, and Geometry From an Algebraic Perspective — rather than in isolation.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

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Standard 3: Geometry

SEVENTH GRADE

Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 2: Measurement and Estimation – The student estimates, measures, and uses measurement formulas in a variety of situations.

Seventh Grade Knowledge Base Indicators	Seventh Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. determines and uses rational number approximations (estimations) for length, width, weight, volume, temperature, time, perimeter, and area using standard and nonstandard units of measure (2.4.K1a) (\$). 2. selects and uses measurement tools, units of measure, and level of precision appropriate for a given situation to find accurate rational number representations for length, weight, volume, temperature, time, perimeter, area, and angle measurements (2.4.K1a) (\$). 3. converts within the customary system and within the metric system (2.4.K1a). 4. ▲ knows and uses perimeter and area formulas for circles, squares, rectangles, triangles, and parallelograms (2.4.K1h). 5. finds perimeter and area of two-dimensional composite figures of circles, squares, rectangles, and triangles (2.4.K1h). 6. ▲ uses given measurement formulas to find (2.4.K1h): <ol style="list-style-type: none"> a. surface area of cubes, b. volume of rectangular prisms. 7. finds surface area of rectangular prisms using concrete objects (2.4.K1h). 8. uses appropriate units to describe rate as a unit of measure (2.4.K1a), e.g., miles per hour. 9. finds missing angle measurements in triangles and quadrilaterals (2.4.K1h). 	<p>The student...</p> <ol style="list-style-type: none"> 1. solves real-world problems by (\$): <ol style="list-style-type: none"> a. converting within the customary and metric systems (2.4.A1a), e.g., James added 30 grams of sand to his model boat that weighed 2 kg. With the sand included, what is the total weight of his boat in kg? b. finding perimeter and area of circles, squares, rectangles, triangles, and parallelograms (2.4.A1h), e.g., the dimensions of a room are 22' x 12'. What is the total length of wallpaper border needed? c. ▲ ■ finding perimeter and area of two-dimensional composite figures of squares, rectangles, and triangles (2.4.A1h), e.g., the front of a barn is rectangular in shape with a height of 10 feet and a width of 48 feet. Above the rectangle is a triangle that is 7 feet high with sides 25 feet long. What is the area of the front of the barn? d. using appropriate units to describe rate as a unit of measure (2.4.A1a), e.g., a person traveled 20 miles in 10 minutes. What is the rate of travel? The answer could be 2 miles per minute or 120 miles per hour. e. finding missing angle measurements in triangles and quadrilaterals (2.4.A1h), e.g., a fenced pasture is a quadrilateral with angles of 30°, 120°, and 90° degrees. What is the measure of the fourth angle? f. applying various measurement techniques (selecting and using measurement tools, units of measure, and level of precision) to find accurate rational number representations for length, weight, volume, temperature, time, perimeter, and area appropriate to a given situation (2.4.A1a).

7-20
January 31, 2004

▲ – Assessed Indicator on the Objective Assessment

■ – Assessed Indicator on the Optional Constructed Response Assessment

N – Noncalculator

(\$) – Financial Literacy

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	<p>2. estimates to check whether or not measurements or calculations for length, width, weight, volume, temperature, time, perimeter, and area in real-world problems are reasonable and adjusts original measurement or estimation based on additional information (a frame of reference) (2.4.A1a) (\$), e.g., students estimate the weight of their book in grams. Then the weight of their calculator is measured in grams. Students then adjust their estimate.</p>
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Teacher Notes: The term *geometry* comes from two Greek words meaning “earth measure.” **Measurement** provides the tools required to apply geometric concepts in the real-world. **Estimation in measurement** is defined as making guesses as to the exact measurement of an object without using any type of measurement tool. Estimation helps students develop a relationship between the different sizes of units of measure. It helps students develop basic properties of measurement and it gives students a tool to determine whether a given measurement is reasonable.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a (\$). The National Standards in Personal Finance are included in the Appendix.

Standard 3: Geometry

SEVENTH GRADE

Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 3: Transformational Geometry – The student recognizes and performs transformations on two- and three-dimensional geometric figures in a variety of situations.

Seventh Grade Knowledge Base Indicators	Seventh Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. identifies, describes, and performs single and multiple transformations [reflection, rotation, translation, reduction (contraction/shrinking), enlargement (magnification/growing)] on a two-dimensional figure (2.4.K1a). 2. identifies three-dimensional figures from various perspectives (top, bottom, sides, corners) (2.4.K1a). 3. draws three-dimensional figures from various perspectives (top, bottom, sides, corners) (2.4.K1a). 4. generates a tessellation (2.4.K1a). 	<p>The student...</p> <ol style="list-style-type: none"> 1. describes the impact of transformations [reflection, rotation, translation, reduction (contraction/shrinking), enlargement (magnification/growing)] on the perimeter and area of squares and rectangles (2.4.A1a); e.g., when a square is rotated, the perimeter and area stays the same, however, when the length of the sides of a square are tripled, the perimeter triples, and the area is 9 times bigger. 2. investigates congruency and similarity of geometric figures using transformations (2.4.A1h). 3. ▲ ■ determines the actual dimensions and/or measurements of a two-dimensional figure represented in a scale drawing (2.4.A1i).

Teacher Notes: Transformational geometry is another way to investigate and interpret geometric figures by moving every point in a plane figure to a new location. To help students form images of shapes through different transformations, students can use concrete objects, figures drawn on graph paper, mirrors or other reflective surfaces, or appropriate technology. Some transformations, like translations, reflections, and rotations, do not change the figure itself. Other transformations like reduction (contraction/shrinking) or enlargement (magnification/growing) change the size of a figure, but not the shape (congruence vs. similarity).

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

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Standard 3: Geometry

SEVENTH GRADE

Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

Benchmark 4: Geometry From An Algebraic Perspective – The student relates geometric concepts to a number line and a coordinate plane in a variety of situations.

Seventh Grade Knowledge Base Indicators	Seventh Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. finds the distance between the points on a number line by computing the absolute value of their difference (2.4.K1a). 2. uses all four quadrants of a coordinate plane to (2.4.K1g): <ol style="list-style-type: none"> a. identify in which quadrant or on which axis a point lies when given the coordinates of a point, b. plot points, c. identify points, d. list through five ordered pairs of a given line. 3. uses a given linear equation with whole number coefficients and constants and a whole number solution to find the ordered pairs, organize the ordered pairs using a T-table, and plot the ordered pairs on the coordinate plane (2.4.K1e-g). 4. examines characteristics of two-dimensional figures on a coordinate plane using various methods including mental math, paper and pencil, concrete objects, and graphing utilities or other appropriate technology (2.4.A1g). 	<p>The student...</p> <ol style="list-style-type: none"> 1. represents and/or generates real-world problems using a coordinate plane to find (2.4.A1g-h): <ol style="list-style-type: none"> a. perimeter of squares and rectangles; e.g., using a coordinate plane Jack started at the school (1, 2), traveled to the post office (3 ½, 2), then went by the fire station (3 ½, 3), then visited the park (1, 3), and finally returned to the school. Determine the distance Jack traveled. b. circumference (perimeter) of circles, e.g., Jane jogs on a circular track with a radius of 100 feet. How far would she jog in one lap? c. area of circles, parallelograms, triangles, squares, and rectangles; e.g., if the sprinkler head is centered at (3, 4) and we know it reaches point (3, - 2) on the coordinate plane, determine the area being watered.

Teacher Notes: A **number line** (a mathematical model) is a diagram that represents numbers with equal distances marked off as points on a line, and is an example of one-to-one correspondence (a relation). A number line can be used as a visual representation of numbers and operations. In addition, a number line used horizontally and vertically is a precursor to the coordinate plane; and the distance between two numbers on a number line is a precursor to absolute value.

A **coordinate plane** (coordinate grid) consists of a horizontal number line called the x-axis and a vertical number line called the y-axis. These two lines intersect at a point called the origin. The x-axis and the y-axis divide the plane into four sections called quadrants. Any point on the coordinate plane can be named with two numbers called coordinates. The first number is the x-coordinate. The second number is the y-coordinate. Since the pair is always named in order (first x, then y), it is called an ordered pair.

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

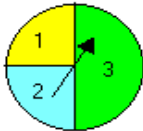
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Standard 4: Data

SEVENTH GRADE

Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.

Benchmark 1: Probability – The student applies the concepts of probability to draw conclusions, generate convincing arguments, and make predictions and decisions including the use of concrete objects in a variety of situations.

Seventh Grade Knowledge Base Indicators	Seventh Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. finds the probability of a compound event composed of two independent events in an experiment or simulation (2.4.K1i) (\$). 2. explains and gives examples of simple or compound events in an experiment or simulation having probability of zero or one. 3. uses a fraction, decimal, and percent to represent the probability of (2.4.K1c): <ol style="list-style-type: none"> a. a simple event in an experiment or simulation; b. a compound event composed of two independent events in an experiment or simulation. 4. finds the probability of a simple event in an experiment or simulation using geometric models (2.4.K1i), e.g., using spinners or dartboards, what is the probability of landing on 2? The answer is $\frac{1}{4}$, .25, or 25%. <div style="text-align: center;">  </div>	<p>The student...</p> <ol style="list-style-type: none"> 1. conducts an experiment or simulation with a compound event composed of two independent events including the use of concrete objects; records the results in a chart, table, or graph; and uses the results to draw conclusions and make predictions about future events (2.4.A1j-k). 2. analyzes the results of an experiment or simulation of a compound event composed of two independent events to draw conclusions, generate convincing arguments, and make predictions and decisions in a variety of real-world situations (2.4.A1j-k). 3. compares results of theoretical (expected) probability with empirical (experimental) probability in an experiment or situation with a compound event composed of two simple independent events and understands that the larger the sample size, the greater the likelihood that the experimental results will equal the theoretical probability(2.4.A1j). 4. makes predictions based on the theoretical probability of a simple event in an experiment or simulation (2.4.A1j).

Teacher Notes: Ideas from **probability** reinforce concepts in the other Standards, especially Number and Computation and Geometry. Students need to develop an intuitive concept of chance – whether or not something is unlikely or likely to happen. Probability experiences should be addressed through the use of concrete objects (process models); spinners, number cubes, or dartboards (geometric models); and coins (money models). Probabilities are ratios, expressed as fractions, decimals, or percents, determined by considering results or outcomes of experiments. Some examples of uses of probability in every day life include: There is a 50% chance of rain today. What is the probability that the team will win every game?

Mathematical models such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, ... with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

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7-27
January 31, 2004

▲ – Assessed Indicator on the Objective Assessment

■ – Assessed Indicator on the Optional Constructed Response Assessment

N – Noncalculator

(§) – Financial Literacy

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Standard 4: Data

SEVENTH GRADE

Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.

Benchmark 2: Statistics – The student collects, organizes, displays, and explains numerical (rational numbers) and non-numerical data sets in a variety of situations with a special emphasis on measures of central tendency.

Seventh Grade Knowledge Base Indicators	Seventh Grade Application Indicators
<p>The student...</p> <ol style="list-style-type: none"> 1. ▲ organizes, displays, and reads quantitative (numerical) and qualitative (non-numerical) data in a clear, organized, and accurate manner including a title, labels, categories, and rational number intervals using these data displays (2.4.K1j) (§): <ol style="list-style-type: none"> a. frequency tables; b. bar, line, and circle graphs; c. Venn diagrams or other pictorial displays; d. charts and tables; e. stem-and-leaf plots (single); f. scatter plots; g. box-and-whiskers plots. 2. selects and justifies the choice of data collection techniques (observations, surveys, or interviews) and sampling techniques (random sampling, samples of convenience, or purposeful sampling) in a given situation. 3. conducts experiments with sampling and describes the results. 4. determines the measures of central tendency (mode, median, mean) for a rational number data set (2.4.K1a) (§). 5. identifies and determines the range and the quartiles of a rational number data set (2.4.K1a) (§). 6. identifies potential outliers within a set of data by inspection rather than formal calculation (2.4.K1a) (§), e.g., consider the data set of 1, 100, 101, 120, 140, and 170; the outlier is 1. 	<p>The student...</p> <ol style="list-style-type: none"> 1. uses data analysis (mean, median, mode, range) of a rational number data set to make reasonable inferences and predictions, to analyze decisions, and to develop convincing arguments from these data displays (2.4.A1k) (§): <ol style="list-style-type: none"> a. frequency tables; b. bar, line, and circle graphs; c. Venn diagrams or other pictorial displays; d. charts and tables; e. stem-and-leaf plots (single); f. scatter plots; g. box-and-whiskers plots. 2. explains advantages and disadvantages of various data displays for a given data set (2.4.A1k) (§). 3. ▲ recognizes and explains (2.4.A1k): <ol style="list-style-type: none"> a. ■ misleading representations of data; b. the effects of scale or interval changes on graphs of data sets. 4. determines and explains the advantages and disadvantages of using each measure of central tendency and the range to describe a data set (2.4.A1a) (§).

Teacher Notes: Graphs (data displays) are pictorial representations of mathematical relationships, are used to tell a story, and are an important part of statistics. When a graph is made, the axes and the scale (numbers running along a side of the graph) are chosen for a reason. The difference between numbers from one grid line to another is the **interval**. The interval will depend on the lowest and highest values in the data set. Emphasizing the importance of using equal-sized pictures or intervals is critical to ensuring that the data display is accurate.

Graphs take many forms:

- bar graphs and pictographs compare discrete data,
- frequency tables show how many times a certain piece of data occurs,
- circle graphs (pie charts) model parts of a whole,
- line graphs show change over time,
- Venn diagrams show relationships between sets of objects,
- line plots show frequency of data on a number line,
- single stem-and-leaf plots (closely related to line plots except that the number line is usually vertical and digits are used rather than x's) show frequency distribution by arranging numbers (stems) on the left side of a vertical line with numbers (leaves) on the right side,
- scatter plots show the relationship between two quantities, and
- box-and-whisker plots are visual representations of the five-number summary – the median, the upper and lower quartiles, and the least and greatest values in the distribution – therefore, the center, the spread, and the overall range are immediately evident by looking at the plot.

Two important aspects of data are its *center* and its *spread*. The mean, median, and mode are **measures of central tendency** (averages) that describe where data are centered. Each of these measures is a single number that describes the data. However, each does it slightly differently. The range describes the spread (dispersion) of data. The easiest way to measure spread is the range, the difference between the greatest and the least values in a data set. Quartiles are boundaries that break the data into fourths.

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7-29
January 31, 2004

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