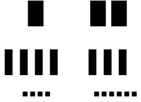
## Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

### Benchmark 1: Number Sense – The student demonstrates number sense for whole numbers, fractions, decimals, and money using concrete objects in a variety of situations.

| Third Grade Knowledge Base Indicators | Third Grade Application Indicators |
|---|---|
| The student…<br>1. knows, explains, and represents **($)**:<br>a. whole numbers from 0 through 10,000 (2.4.K1a-b)<br>b. fractions greater than or equal to zero (halves, fourths, thirds, eighths, tenths, sixteenths) (2.4.K1c) **($)**;<br>c. decimals greater than or equal to zero through tenths place (2.4.K1c).<br>2. compares and orders:<br>a. ▲ ■ whole numbers from 0 through 10,000 with and without the use of concrete objects (2.4.K1a-b) **($)**;<br>b. fractions greater than or equal to zero with like denominators (halves, fourths, thirds, eighths, tenths, sixteenths) using concrete objects (2.4.K1a,c);<br>c. decimals greater than or equal to zero through tenths place using concrete objects (2.4.K1a-c).<br>3. ▲ knows, explains, and uses equivalent representations including the use of mathematical models for:<br>a. addition and subtraction of whole numbers from 0 through 1,000 (2.4.K1a-b) **($)**, e.g., 144 + 236 = 300 + 80<br><br>$100 $10 $10<br>$100 $10 $10<br>$100 $10 $10<br>$10 $10 | The student…<br>1. solves real-world problems using equivalent representations and concrete objects to **($)**:<br>a. compare and order whole numbers from 0 through 5,000 (2.4.A1a-b), e.g., using base ten blocks, represent the total school attendance for a week; then represent the numbers using digits and compare and order in different ways;<br>b. add and subtract whole numbers from 0 through 1,000 and when used as monetary amounts (2.4.A1a,d) **($)**, e.g., use real money to show at least 2 ways to represent $10.42; then subtract the cost of a book purchases at the school's book fair from $10.42 (the amount you have earned and can spend).<br>2. determines whether or not solutions to real-world problems that involve the following are reasonable **($)**.<br>a. whole numbers from 0 through 1,000 (2.4.A1a-b), e.g., a student says that there are 1,000 students in grade 3 at her school, is this reasonable?<br>b. fractions greater than or equal to zero (halves, fourths, thirds, eighths, tenths, sixteenths) (2.4.A1a,c); e.g., you ate ½ of a sandwich and a friend ate ¾ of the same sandwich; is this reasonable?<br>c. decimals greater than or equal to zero when used as monetary amounts (2.4.A1d), e.g., a pack of chewing gum costs what amount - $62 $.75 9¢ $75.00 750¢? Is this reasonable?; |

**▲– Assessed Indicator on the Objective Assessment**
**■ – Assessed Indicator on the Optional Constructed Response Assessment**
**N – Noncalculator**
**($) – Financial Literacy** **THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

| | |
|---|---|
| b. multiplication using the basic facts through the 5s and the multiplication facts of the 10s (2.4.K1a), e.g., 3 x 2 can be represented as 4 + 2 or as an array, X X X<br>X X X; | 3. determines the amount of change owed through $100.00 (2.4.A1d), e.g., school supplies cost $12.37. What was the amount of change received after giving the clerk $20.00? To solve, $20.00 – $12.37 = $7.63 (the change). |
| c. addition and subtraction of money (2.4.K1d) **($)**, e.g., three half dollars equals 50¢ + 50¢ + 50¢ or 50¢ + 100¢.<br>4. **▲N** determines the value of mixed coins and bills with a total value of $50 or less (2.1.K1d) **($)**. | |

**Teacher Notes: Number sense** refers to one's ability to reason with numbers and to work with numbers in a flexible way. The ability to compute mentally, to estimate based on understanding of number relationships and magnitudes, and to judge reasonableness of answers are all involved in number sense.

When we say that someone has good number sense, we mean that he or she possesses a variety of abilities and understandings that include an awareness of the relationships between numbers, an ability to represent numbers in a variety of ways, a knowledge of the effects of operations, and an ability to interpret and use numbers in real-world counting and measurement situations. Such a person predicts with some accuracy the result of an operation and consistently chooses appropriate measurement units. This "friendliness with numbers" goes far beyond mere memorization of computational algorithms and number facts; it implies an ability to use numbers flexibly, to choose the most appropriate representation of a number for a given circumstance, and to recognize when operations have been correctly performed. (Number Sense and Operations: Addenda Series, Grades K-6, NCTM, 1993)

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a **($)**. The National Standards in Personal Finance are included in the Appendix.

## Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

### Benchmark 2: Number Systems and Their Properties – The student demonstrates an understanding of whole numbers with a special emphasis on place value and recognizes, uses, and explains the concepts of properties as they relate to whole numbers, fractions, decimals, and money in a variety of situations.

| Third Grade Knowledge Base Indicators | Third Grade Application Indicators |
|---|---|
| The student…<br>1. identifies, reads, and writes numbers using numerals and words from tenths place through ten thousands place (2.4.K1a-b) **($)**, e.g., sixty-four thousand, three hundred eighty and five tenths is written in numerical form as 64,380.5.<br>2. identifies, models, reads, and writes numbers using expanded form from tenths place through ten thousands place (2.4.K1b), e.g., 56,277.3 = (5 x 10,000) + (6 x 1,000) + (2 x 100) + (7 x 10) + (7 x 1) + (3 x .1) = 50,000 + 6,000 + 200 + 70 + 7 + .3.<br>3. classifies various subsets of numbers as whole numbers, fractions (including mixed numbers), or decimals (2.4.K1a-c, 2.4.K1i)<br>4. identifies the place value of various digits from tenths to one hundred thousands place (2.4.K1b) **($)**.<br>5. identifies any whole number through 1,000 as even or odd (2.4.K1a).<br>6. **uses** the **concepts** of these properties with whole numbers from 0 through 100 and demonstrates their meaning including the use of concrete objects (2.4.K1a) **($)**:<br>a. commutative properties of addition and multiplication, e.g., 7 + 8 = 8 + 7 or 3 x 6 = 6 x 3;<br>b. zero property of addition (additive identity), e.g., 4 + 0 = 4;<br>c. property of one for multiplication (multiplicative identity), 1 x 3 = 3;<br>d. associative property of addition, e.g., (3 + 2) + 4 = 3 + (2 + 4);<br>e. symmetric property of equality applied to addition and multiplication, e.g., 100 = 20 + 80 is the same as 20 + 80 = 100 and 3 x 4 = 12 is the same as 12 = 3 x 4; | The student…<br>1. solves real-world problems with whole numbers from 0 through 100 using place value models, money, and the concepts of these properties to explain reasoning (2.4.A1a-b,d) **($)**:<br>a. commutative property of addition, e.g., a student has a dime, a nickel, and a quarter to purchase a pencil; he/she totals the amount of the coins to see whether or not there is enough money; the student could count the quarter, nickel, and dime as 25¢ + 5¢ + 10¢ or as 25¢ + 10¢ + 5¢ because adding in any order does not change the sum;<br>b. zero property of addition, e.g., a student has 6 marbles in one pocket and none in the other, so all together there are: 6 + 0 = 6;<br>c. associative property of addition, e.g., a student has two dimes and a quarter; there are 2 ways to group the coins to find the total: 10¢ (dime) + 10¢ (dime) = 20¢, then add the quarter, 20¢ + 25¢ (quarter) = 45¢ or 10¢ (dime) + 25¢ (quarter) = 35¢, then add the other dime to 35¢ and 35¢ + 10¢ = 45¢ or (D + D) + Q = D + (D + Q) using coins or money models.<br>2. performs various computational procedures with whole numbers from 0 through 100 using the concepts of these properties and explains how they were used (2.4.A1a-b):<br>a. commutative property of multiplication, e.g., given 4 x 6, the student says: I know that 4 x 6 is 24 because I know 6 x 4 is 24 and multiplying in any order gets the same answer; |

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|---|---|
| f. zero property of multiplication, e.g., 9 x 0 = 0 or 0 x 32 = 0.<br>7. divides whole numbers from 0 through 99,999 into groups of 10,000s; 1,000s; 100s; 10s, and 1s using base ten models (2.4.K1b). | b. zero property of multiplication without computing, e.g., 7 x 3 x 4 x 0 x 5 = □, the student says: I know the answer (product) is zero because no matter how many factors you have, when you multiply with a 0, the product is zero;<br>c. associative property of addition, e.g., 9 + 8 could be solved as 1 + (8 + 8) or (1 + 8) + 8, the student says: I don't know 9 + 8, but I know my doubles (8 + 8), so I made the 9 into 1 + 8 and added 8 + 8 and then added 1 more to make 17. |

**Teacher Notes:** From the Mathematics Dictionary and Handbook (Nichols Schwartz Publishing, 1999), **property** as a mathematical term means a characteristic (an attribute) of a number, geometric shape, mathematical operation, equation, or inequality. To give an example:

- Property of a number: 8 is divisible by 2.
- Property of a geometric shape: Each of the four sides of a square is of the same length.
- Property of an operation: Addition is commutative. For all numbers x and y, x + y = y + x.
- Property of an equation: For all numbers a, b, and c, if a = b, then a + c = b + c.
- Property of an inequality: For all numbers a, b, and c, if a > b, then a – c > b – c.

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

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## Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

### Benchmark 3: Estimation – The student uses computational estimation with whole numbers, fractions, and money in a variety of situations.

| Third Grade Knowledge Base Indicators | Third Grade Application Indicators |
|---|---|
| The student…<br>1. estimates whole numbers quantities from 0 through 1,000; fractions (halves, fourths); and monetary amounts through $500 using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.K1a-d) **($)**.<br>2. uses various estimation strategies to estimate using whole number quantities from 0 through 1,000 and explains the process used (2.4.K1a) **($)** e.g., 362 rounded to the nearest ten is 360 and 362 rounded to the nearest hundred is 400. Using front-end estimation, 362 is about 300 or 400 depending on the context of the problem. Using a "nice" number, 362 is about 350 because of the benchmark number – 350, since 350 is the halfway point between 300 and 400.<br>3. recognizes and explains the difference between an exact and an approximate answer (2.4.K1a), e.g., when asked how many students are in a classroom, an exact answer could be 24. Whereas, an approximate answer could be 20 since 24 could be rounded down to the nearest ten (**underestimated**) or rounded up to 30 (**overestimated**). | The student…<br>1. adjusts original whole number estimate of a real-world problem using numbers from 0 through 1,000 based on additional information (a frame of reference) (2.4.A1a) **($)**, e.g., if given a pint container and told the number of marbles it has in it, the student would estimate the number of marbles in a quart container.<br>2. estimates to check whether or not the result of a real-world problem using whole numbers from 0 through 1,000 and monetary amounts through $500 is reasonable and makes predictions based on the information (2.4.A1a-b,d) **($)**, e.g., at the movies, you bought popcorn for $2.35 and a soda for $2.50; and then paid $4.50 for a ticket. Is it reasonable to say you spent $10? How much will you need to save to go to the movies once a week for the next month?<br>3. selects a reasonable magnitude from three given quantities based on a familiar problem situation and explains the reasonableness of the results (2.4.A1a), e.g., about how many students are in my class today – 2, 20, 200?<br>4. determines if a real-world problem with whole numbers from 0 through 1,000 calls for an exact or approximate answer and performs the appropriate computation using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.A1a) **($)**. |

**Teacher Notes: Estimate**, as a verb, means to make an educated guess based on information in a problem or to give an answer close to the exact number. Estimation is used when an exact answer is not needed, as in many real-life situations for which "ballpark" computations are acceptable. Good number sense enables one to estimate a quantity, estimate a measure, or estimate an answer.

**Estimation** serves as an important companion to computation. It provides a tool for judging the reasonableness of computational methods including mental math, paper and pencil, concrete objects, and appropriate technology. However, being able to compute does not automatically lead to an ability to estimate or judge reasonableness of answers. Frequent modeling by the teacher helps students develop a range of estimation strategies. Students should be encouraged to frequently explain their thinking as they estimate. As with exact computation, sharing estimation strategies allows students access to others' thinking and provides opportunities for class discussion. Identifying the estimation strategy by name is not critical; however, explaining the thinking behind the strategy to make a valid estimation is important. (Principles and Standards for School Mathematics, NCTM, 2000)

**Mental math** and **estimation** are distinct but related mathematical skills. Proficiency in mental math contributes to increased skill in estimation. In order for students to become more familiar with estimation, teachers should introduce estimation with examples where rounded or estimated numbers are used. Emphasis should be placed on real-world examples where only estimation is required, e.g., About how many hours do you sleep a night? Using the language of estimation is important, so students begin to realize that a variety of estimates (answers) are possible.
In addition, when students are taught specific estimation strategies, they develop mental math and estimation skills easier. Estimation strategies include front-end with adjustment, compatible "nice" numbers, clustering, or special numbers.

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## Standard 1: Number and Computation – The student uses numerical and computational concepts and procedures in a variety of situations.

### Benchmark 4: Computation – The student models, performs, and explains computation with whole numbers and money including the use of concrete objects in a variety of situations.

| Third Grade Knowledge Base Indicators | Third Grade Application Indicators |
|---|---|
| The student…<br>1. computes with efficiency and accuracy using various computational methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.K1a) **($)**.<br>2. **N** states and uses with efficiency and accuracy the multiplication facts through the 5s and the multiplication facts of the 10s and corresponding division facts (2.4.K1a) **($)**.<br>3. skip counts (multiples) by 2s, 3s, 4s, 5s, and 10s (2.4.K1a).<br>4. **N** performs and explains these computational procedures:<br>a. adds and subtracts whole numbers from 0 through 10,000 (2.4.K1a-b);<br>b. multiplies whole numbers when one factor is 5 or less and the other factor is a multiple of 10 through 1,000 with or without the use of concrete objects (2.4.K1a-b), e.g., 400 x 3 = 120 or 70 x 5 = 350;<br>c. adds and subtracts monetary amounts using dollar and cents notation through $500.00 (2.4.K1d) **($)**, e.g., $47.07 + $356.96 = $404.03.<br>5. fair shares/measures out (divides) a total amount through 100 concrete objects into equal groups (2.4.K1a-b), e.g., fair sharing 52 pieces of candy with 8 friends resulting in eight groups of 6 with four pieces left over or measuring out into groups of eight 52 pieces of candy with four pieces left over.<br>6. explains the relationship between addition and subtraction (2.4.K1a-b) **($)**.<br>7. ▲■ **N** identifies multiplication and division fact families through the 5s and the multiplication and division fact families of the 10s (2.4.K1a), e.g., when given 6 x □ = 18, the student recognizes the remaining members of the fact family. | The student…<br>1. ▲**N** solves one-step real-world addition or subtraction problems with **($)**:<br>a. whole numbers from 0 through 10,000 (2.4.A1a-b), e.g., for the food drive, the school collected 564 cans (cylinders) and 297 boxes (rectangular prisms). How many items did they collect in all? This problem could be solved with base 10 models: by adding 500 + 200 (700), 60 + 90 (150), and 4 + 7 (11), so 700+ 150 + 11= 861; by adding 564 + 300 (864) and 297 is 3 less than 300, so 864 – 3 = 861; or by using the traditional algorithm;<br>b. monetary amounts using dollar and cents notation through $500.00 (2.4.A1a-b,d), e.g., you are shopping for a new bicycle; at The Bike Store, the bike you want is $189.69 and at Sports for All, it is $162.89. How much will you save by buying the bike at Sports for All?<br>2. **N** generates a family of multiplication and division facts through the 5s (2.4.A1a), e.g., if the student writes 5 x 9 = 45, the remaining facts generated are: 9 x 5 = 45, 45 ÷ 5 = 9, 45 ÷ 9 = 5. |

**▲– Assessed Indicator on the Objective Assessment**
**■ – Assessed Indicator on the Optional Constructed Response Assessment**
**N – Noncalculator**
**($) – Financial Literacy**

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8. reads and writes horizontally, vertically, and with different operational symbols the same addition, subtraction, multiplication, or division expression, e.g., 4 • 6 is the same as 4 x 6 or 4(6) or 6 and 10 divided by 2 is the same as 10 ÷ 2 or $\frac{10}{2.}$ $\underline{\text{x 4}}$

**Teacher Notes: Efficiency and accuracy** means that students are able to compute single-digit numbers with fluency. Students increase their understanding and skill in single-digit addition and subtraction by developing relationships within addition and subtraction combinations and by counting on for addition and counting up for subtraction and unknown-addend situations. Students learn basic number combinations and develop strategies for computing that makes sense to them. Through class discussions, students can compare the ease of use and ease of explanation of various strategies. In some cases, their strategies for computing will be close to conventional algorithms; in other cases, they will be quite different. Many times, students' invented approaches are based on a sound understanding of numbers and operations, and these invented approaches often can be used with efficiency and accuracy. (Principles and Standards for School Mathematics, NCTM, 2000)

The definition of computation is finding the standard representation for a number. For example, 6 + 6, 4 x 3, 17 – 5, and 24 ÷ 2 are all representations for the standard representation of 12. **Mental math** is mentally finding the standard representation for a number – calculating in your head instead of calculating using paper and pencil or technology. One of the main reasons for teaching mental math is to help students determine if a computed/calculated answer is reasonable; in other words, using mental math to estimate to see if the answer makes sense. Students develop mental math skills easier when they are taught specific strategies. Mental math strategies include counting on, doubling numbers (doubles), making ten, and compatible numbers.

Regrouping refers to the reorganization of objects. In computation, **regrouping** is based on a "partitioning to multiples of ten" strategy. For example, 46 + 7 could be solved by partitioning 46 into 40 and 6, then 40 + (6 + 7) = 40 + 13 (and then 13 is partitioned into 10 and 3) which then becomes (40 + 10) + 3 becomes 50 + 3 = 53 *or* 7 could be partitioned as 4 and 3, then 46 + 4 (bridging through 10) = 50 and 50 + 3 = 53. Before algorithmic procedures are taught, an understanding of "what happens" must occur. For this to occur, instruction should involve the use of structured manipulatives. To emphasize the role of the base ten numeration system in algorithms, some form of expanded notation is recommended. During instruction, each child should have a set of manipulatives to work with rather than sit and watch demonstrations by the teacher. Some additional computational strategies include *doubles plus one or two* (i.e., 6 + 8, 6 and 6 are 12, so the answer must be 2 more or 14), *compensation* (i.e., 9 + 7, if one is taken away from 9, it leaves 8; then that one is given to 7 to make 8, then 8 + 8 = 16), *subtracting through ten* (i.e., 13 – 5, 13 take away 3 is 10, then take 2 more away from 10 and that is 8), and *nine is one less than ten* (i.e., 9 + 6, 10 and 6 are 16, and 1 less than 16 makes 15). (Teaching Mathematics in Grades K-8: Research Based Methods, ed. Thomas R. Post, Allyn and Bacon, 1988)

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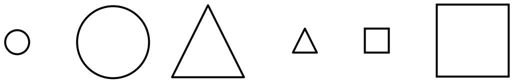
## Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

### Benchmark 1: Patterns – The student recognizes, describes, extends, develops, and explains relationships in patterns using concrete objects in a variety of situations.

| Third Grade Knowledge Base Indicators | Third Grade Application Indicators |
|---|---|
| The student…<br>1. uses concrete objects, drawings, and other representations to work with **types** of patterns (2.4.K1a):<br>a. repeating patterns, e.g., an AB pattern is like 1-2, 1-2, …; an ABC pattern is like dog-horse-pig, dog-horse-pig, …; an AAB pattern is like ↑↑→, ↑↑→, …;<br>b. growing patterns, e.g., 1, 4, 7, 10, …<br>2. uses these **attributes** to generate patterns:<br>a. counting numbers related to number theory (2.4.K1a), e.g., evens, odds, or multiples through the 5s;<br>b. whole numbers that increase or decrease (2.4.K1a) **($)**,e.g., 3, 6, 9, …; 20, 15, 10, …;<br>c. geometric shapes including one attribute change (2.4.K1f), e.g., ■-☐-Δ-▲, ■-☐-Δ-▲, ■-☐-Δ-▲,… where the pattern is filled-in square, square, triangle, filled-in triangle, …; or when using attribute blocks the change is size only, then shape only, … such as<br> …;<br>d. measurements (2.4.K1a), e.g., 1 ft, 2 ft, 3 ft, …; 3 lbs, 6 lbs, 9 lbs; or 2 cups, 4 cups, 6 cups, …;<br>e. money and time (2.4.K1a,d) **($)**, e.g., $.25, $.50, $.75, … or 1:05 p.m., 1:10 p.m., 1:15 p.m., …;<br>f. things related to daily life (2.4.K1a), e.g., water cycle, food cycle, or life cycle; | The student…<br>1. generalizes the following patterns using a written description:<br>a. counting numbers related to number theory (2.4.A1a);<br>b. whole number patterns (2.4.A1a) **($)**,<br>c. patterns using geometric shapes (2.4.A1f),<br>d. measurement patterns (2.4.A1a),<br>e. money and time patterns (2.4.A1a,d) **($)**,<br>f. patterns using size, shape, color, texture, or movement (2.4.A1a).<br>2. ▲ recognizes multiple representations of the same pattern (2.4.A1a) e.g., the ABC pattern could be represented by clap, snap, stomp, …; red, green, yellow, …; tricycle, bicycle, unicycle, …; or 3, 2, 1, … |

| | |
|---|---|
| g. things related to size, shape, color, texture, or movement (2.4.K1a), e.g., red-green, red-green, red-green, …; snapping fingers; clapping hands; stomping feet; or tossing a bean bag over the head, under the leg, and behind the back (kinesthetic patterns).<br>3. identifies, states, and continues a pattern presented in various formats including numeric (list or table), visual (picture, table, or graph), verbal (oral description), kinesthetic (action), and written (2.4.K1a) **($)**.<br>4. generates:<br>a. repeating patterns (2.4.K1a),<br>b. growing (extending) patterns (2.4.K1a),<br>c. patterns using function tables (input/output machines, T-tables) (2.4.K1e). | |

**Teacher Notes:** Working with **patterns** is an important process in the development of mathematical thinking. Patterns can be based on geometric attributes (shapes, regions, angles); measurement attributes (color, texture, length, weight, volume, number); relational attributes (proportion, sequence, functions); and affective attributes (values, likes, dislikes, familiarity, heritage, culture). (Learning to Teach Mathematics, Randall J. Souviney, Macmillan Publishing Company, 1994)

This process (working with patterns) can be used to develop or deepen understandings of important concepts in number theory, whole numbers, measurement, geometry, probability, and functions. Working with patterns provides opportunities for students to recognize, describe, extend, develop, and explain.

**Number theory** is the study of the properties of the counting numbers (positive integers), their relationships, ways to represent them, and patterns among them. Number theory includes the concepts of odd and even numbers, factors and multiples, primes and composites, and greatest common factor and least common multiple.

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a **($)**. The National Standards in Personal Finance are included in the Appendix.

## Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

### Benchmark 2: Variables, Equations, and Inequalities – The student uses symbols and whole numbers to solve equations including the use of concrete objects in a variety of situations.

| Third Grade Knowledge Base Indicators | Third Grade Application Indicators |
|---|---|
| The student…<br>1. explains and uses symbols to represent unknown whole number quantities from 0 through 1,000 (2.4.K1a)<br>2. finds the sum or difference in one-step equations with **($)**:<br>a. whole numbers from 0 through 99 (2.4.K1a) e.g., 89 = 76 + y or y – 23 = 32;<br>b. monetary values through a dollar (2.4.K1d), e.g., 25¢ + 10¢ + 5¢ = n.<br>3. finds the unknown in the multiplication and division fact families through the 5s and the 10s (2.4.K1a), e.g., 3 • □ = 4 • 6.<br>4. compares two whole numbers from 0 through 1,000 using the equality and inequality symbols (=, <, >) and their corresponding meanings (is equal to, is less than, is greater than) (2.4.K1a-b) **($)**. | The student…<br>1. represents real-world problems using symbols with one operation and one unknown that (2.4.A1a) **($)**:<br>a. adds or subtracts using whole numbers from 0 through 99, e.g., when asked to represent the number of 3rd graders in a school, students write: 21 + 18 + 19 = □ ;<br>b. multiplies or divides using the basic facts through the 5s and the basic facts of the 10s, e.g., juice comes in packs of 4. How many packs are needed for 32 third-graders? Students could write: 32 ÷ 4 = J.<br>2. generates one-step equations to solve real-world problems with one unknown and a whole number solution that (2.4.A1a) **($)**:<br>a. adds or subtracts using the basic fact families, e.g., when asked to generate a simple equation, a student says: I have 5 dogs and 2 fish. How many pets do I have? This is represented by 5 + 2 = P and to solve for P, add 5 and 2, P = 7.<br>b. multiplies or divides using the basic facts through the 5s and the basic facts of the 10s, e.g., Tom has a sticker book and each page holds 5 stickers. If the same number of stickers is placed on each page, the book will hold 30 stickers. How many pages are in his book? This is represented by 5 x S = 30 or 30 ÷ 5 = S.<br>3. generates (2.4.A1a) **($)**:<br>a. a real-world problem with one operation that matches a given addition equation or subtraction equation using whole numbers from 0 through 99, e.g., given the subtraction equation, 69 – G = 37, the problem could be written: You have 69 guppies and give away some to a friend and have 37 left. How many guppies did you give away?<br>b. a real-world problem with one operation that matches a given multiplication equation or division equation using basic facts |

| | |
|---|---|
| | through the 5s and the basic facts of the 10s, e.g., the problem could be: I have 25 pictures and glue 5 pictures on each page of my album. How many pages will I need to use? The equation: 25/5 = ▲.<br>c. number comparison statements using equality and inequality symbols (=, <, >) for whole numbers from 0 through 100, measurement, and money $, e.g.4 ft 4 in > 4 ft 2 in. |

**Teacher Notes:** Understanding the **concept of variable** is fundamental to algebra. In the early grades, students use various symbols, including letters and geometric shapes, to represent unknown quantities that do or do not vary. Quantities that are not given and do not vary are often referred to as unknowns or missing elements when they appear in equations, e.g., 2 x 4 = Δ.

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

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## Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

### Benchmark 3: Functions – The student recognizes and describes whole number relationships using concrete objects in a variety of situations.

**Third Grade Knowledge Base Indicators**

The student…

1. states mathematical relationships between whole numbers from 0 through 200 using various methods including mental math, paper and pencil, concrete objects, and appropriate technology (2.4.K1a) **($)**, e.g., every time a quarter is added to the amount; 25¢ is added to the total.
2. finds the values and determines the rule with one operation (addition, subtraction) of whole numbers from 0 through 200 using a horizontal or vertical function table (input/output machine, T-table) (2.4.K1e), e.g., using this input/output machine, different student responses might be that the rule is Input minus 10 equals Output, the rule is N – 10, or the rule is subtract 10.

| **Input** | **Output** |
|---|---|
| 92 | 82 |
| 156 | 146 |
| 13 | 3 |
| 113 | 103 |
| ? | 59 |
| 106 | ? |
| ? | ? |
| N | ? |

3. ▲ generalizes numerical patterns using whole numbers from 0 through 200 with one operation (addition, subtraction) by stating the rule using words, e.g., if the sequence is 30, 50, 70, 90, …; in words, the rule is add twenty to the number before.
4. uses a function table (input/output machine, T-table) to identify and plot ordered pairs in the first quadrant of a coordinate plane (2.4.K1a,e).

**Third Grade Application Indicators**

The student…

1. represents and describes mathematical relationships between whole numbers from 0 through 100 using concrete objects, pictures, written descriptions, symbols, equations, tables, and graphs (2.4.A1a) **($)**.
2. finds the rule, states the rule using words, and extends numerical patterns with whole numbers from 0 through 100 (2.4.A1a,e), e.g., at school each student must check out three library books. After the tenth student has checked out, how many total books will have been checked out? A solution using a function table might be:

| **Number of Students** | 1 | 2 | 5 | 10 |
|---|---|---|---|---|
| **Total Number Of Books** | 3 | 6 | 15 | ? |

The rule could be that for every student, add three books or multiply the number of children by three to get the total number of books. Other solutions might be using a pattern to count by three ten times - 3, 6, 9, 12, 15, 18, 21, 24, 27, 30 - or skip count by three ten times.

**Teacher Notes:** **Functions** are relationships or rules in which each member of one set is paired with one, and only one, member of another set (an ordered pair). The concept of function can be introduced using function machines. Any number put in the machine will be changed according to some rule. A record of inputs and corresponding outputs can be maintained in a two-column format. Function tables, input/output machines, and T-tables may be used interchangeably and serve the same purpose.

Function concepts should be developed from **growing patterns**. Each term in a number sequence is related to its position in the sequence – the functional relationship. The pattern – 4, 7, 10, 13, 16, 19, and so on – is an arithmetic sequence *with a difference of 3*. The pattern could be described as *add 3* meaning that 3 must be added to the previous term to find the next. This pattern is explained by using the recursive definition for a sequence. The recursive definition for a sequence is a statement or a set of statements that explains how each successive term in the sequence is obtained from the previous term(s).

In the pattern 1, 4, 9, 16, 25, …, 225; there is *no common difference.* This sequence is not arithmetic or geometric (no common ratio between geometric terms). Neither is it a combination of the two; however, there is a pattern and the missing terms between 25 and 225 can be found. To find the term value, square the number of the term. The next missing terms would be 36, 49, 64, 81, 100, 121, and 144. This pattern is explained by using the explicit formula for a sequence. The explicit formula for a sequence defines a rule for finding each term in the number sequence related to its position in the sequence. In other words, to find the term value, square the number of the term – the 5th term is $5^2$, the 8th term is $8^2$, …

Patterns themselves are not explicit or recursive. The *RULE* for the pattern can be expressed explicitly or recursively and *MOST* patterns can be explained using either format especially *IF* that pattern reflects either an arithmetic sequence or geometric sequence.

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

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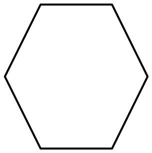
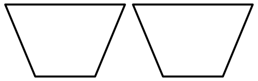
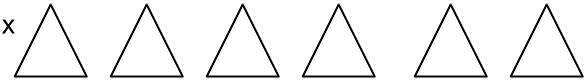
## Standard 2: Algebra – The student uses algebraic concepts and procedures in a variety of situations.

### Benchmark 4: Models – The student develops and uses mathematical models including the use of concrete objects to represent and show mathematical relationships in a variety of situations.

| Third Grade Knowledge Base Indicators | Third Grade Application Indicators |
|---|---|
| The student… | The student… |
| 1. knows, explains, and uses mathematical models to represent mathematical concepts, procedures, and relationships. Mathematical models include: | 1. recognizes that various mathematical models can be used to represent the same problem situation. Mathematical models include: |
| a. process models (concrete objects, pictures, number lines, coordinate planes/grids, hundred charts, measurement tools, multiplication arrays, or division sets) to model computational procedures and mathematical relationships (1.2.K1, 1.2.K.1a, 1.2.K2 1.2.K3, 1.2.K5-6, 1.3.K1-3, 1.4.K1-3, 1.4.K1a-b, 1.4.K5-7, 2.1.K1, 2.1.K2a, 2.1.K2d-g, 2.1.K3, 2.1.K4a-b, 2.2.K1, 2.2.K2, 2.2.K3-4, 2.3.K1, 2.3.K4, 3.2.K1-4, 3.3.K1, 3.4.K1-3, K.2.K3) **($)**; | a. process models (concrete objects, pictures, number lines, coordinate planes/grids, hundred charts, measurement tools, multiplication arrays, or division sets) to model computational procedures and mathematical relationships and to model problem situations (1.2.A1, 1.2.A2a-b, 1.3.A1-4, 1.4.A1a-b, 1.4.A2, 2.1.A1a-b, 2.1.A1d-f, 2.1.A2, 2.2.A1-3, 2.2.A3a-c, 2.3.A1-2, 3.2.A1-3, 3.3.A1-2, 3.4.A1, 4.2.A2) **($)**; |
| b. place value models (place value mats, hundred charts, base ten blocks or unifix cubes) to compare, order, and represent numerical quantities and to model computational procedures (1.1.K1c, 1.1.K2a, 1.1.K2c, 1.1.K3a, 1.2.K1-4, 1.2.K7, 1.3.K1, 1.4.K4a-b, 1.4.K5-6, 2.2.K4) **($)**; | b. place value models (place value mats, hundred charts, base ten blocks, or unifix cubes) to compare, order, and represent numerical quantities and to model computational procedures (1.1.A1a, 1.1.A2a, 1.2.A1-2, 1.3.A2, 1.4.A1a-b) **($)**; |
| c. fraction models (fraction strips or pattern blocks) and decimal models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.K1b, 1.1.K2b-c, 1.2.K3, 1.3.K1) **($)**; | c. fraction models (fraction strips or pattern blocks) and decimal models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.A2b) **($)**; |
| d. money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1K3c, 1.1.K4, 1.3.K1, 1.4.K4c, 2.1.K2e, 2.2.K2b) **($)**; | d. money models (base ten blocks or coins) to compare, order, and represent numerical quantities (1.1.A1b, 1.1.A2c, 1.2.A1, 1.3.A2, 1.4.A1b, 2.1.A1e, 2.2.A3c) **($)**; |
| e. function tables (input/output machines, T-tables) to find numerical relationships (2.1.K4c, 2.3.K2, 2.3.K4) **($)**; | e. function tables (input/output machines, T-tables) to model numerical relationships (2.3.A2) **($)**; |
| | f. two-dimensional geometric models (geoboards, dot paper, pattern blocks, or tangrams) to model perimeter, area, and properties of geometric shapes and three-dimensional geometric models (solids) and real-world objects to compare size and to model attributes of geometric shapes (2.1.A1c, 3.1.A1-3); |

**▲– Assessed Indicator on the Objective Assessment**
**■ – Assessed Indicator on the Optional Constructed Response Assessment**
**N – Noncalculator**
**($) – Financial Literacy**

**THESE STANDARDS ARE ALIGNED ONLY TO THE ASSESSMENTS THAT WILL BEGIN DURING THE 2005-06 SCHOOL YEAR.**

| | |
|---|---|
| f. two-dimensional geometric models (geoboards, dot paper, pattern blocks, or tangrams) to model perimeter, area, and properties of geometric shapes and three-dimensional geometric models (solids) and real-world objects to compare size and to model attributes of geometric shapes (2.1.K2c, 3.1.K1-6, 3.2.K5, 3.3.K2);<br>g. two-dimensional geometric models (spinners), three-dimensional models (number cubes), and process models (concrete objects) to model probability (4.1.K1-2) **($)**;<br>h. graphs using concrete objects, representational objects, or abstract representations, pictographs, frequency tables, horizontal and vertical bar graphs, Venn diagrams or other pictorial displays, line plots, charts, and tables to organize and display data (2.3.K4, 4.1.K2, 4.2.K1a-d, 4.2.K1f-g, 4.2.K2) **($)**;<br>i. Venn diagrams to sort data and show relationships (1.2.K3).<br>2. creates a mathematical model to show the relationship between two or more things, e.g., using pattern blocks, a whole (1) can be represented as<br>a  (1/1) or<br>two  (2/2) or<br>three  (3/3) or<br>six  (6/6). | g. two-dimensional geometric models (spinners), three-dimensional models (number cubes), and process models (concrete objects) to model probability (4.1.A1-2) **($);**<br>h. graphs using concrete objects, representational objects, or abstract representations pictographs, frequency tables, horizontal and vertical bar graphs, Venn diagrams or other pictorial displays, line plots, charts and tables to organize and display data (4.1.A1-2, 4.2.A1a-d, 4.2.A1f-g, 4.2.A3) **($)**;<br>i. Venn diagrams to sort data and show relationships.<br>2. selects a mathematical model that is more useful than other mathematical models in a given situation. |

**Teacher Notes:** For assessment purposes, the mathematical modeling process appropriate to the indicator may be included as part of the item being assessed.

The **mathematical modeling** process involves:

a. selecting key features and relationships within the real-world situation and representing these concepts in mathematical terms through some sort of mathematical model,
b. performing manipulations and mathematical procedures within the mathematical model,
c. interpreting the results of the manipulations within the mathematical model,
d. using these results to make inferences about the original real-world situation.

The use of **mathematical models** is necessary for conceptual understanding. The ways in which mathematical ideas are represented is fundamental to how students understand and use those ideas. As students begin to use multiple representations of the same situation, e.g., comparing the number of boys and girls in the classroom can be represented by lining them up in two different lines. The same situation also can be represented by pictures of the children (pictograph), a bar graph, or by using two different colors of the same manipulative (unifix cubes or color tiles). As students work with the different representations of the same situation, they begin to develop an understanding of the advantages and disadvantages of the various representations/models.

Many **mathematical models** are listed in this benchmark. The indicator lists some of the mathematical models that could be used to teach a concept. Each indicator in this benchmark is linked to other indicators in other benchmarks; those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3. In addition, the indicator in the other benchmarks identifies, in parentheses, the Models' indicator. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models).

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## Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

### Benchmark 1: Geometric Figures and Their Properties – The student recognizes geometric shapes and investigates their properties using concrete objects in a variety of situations.

| Third Grade Knowledge Base Indicators | Third Grade Application Indicators |
|---|---|
| The student…<br>1. recognizes and investigates properties of plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, octagons) using concrete objects, drawings, and appropriate technology (2.4.K1f).<br>2. recognizes, draws, and describes plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, octagons) (2.4.K1f).<br>3. ■ recognizes the solids (cubes, rectangular prisms, cylinders, cones, spheres) (2.4.K1f).<br>4. ▲ recognizes and describes the square, triangle, rhombus, hexagon, parallelogram, and trapezoid from a pattern block set (2.4.K1f).<br>5. recognizes and describes a quadrilateral as any four-sided figure (2.4.K1f).<br>6. determines if geometric shapes and real-world objects contain line(s) of symmetry and draws the line(s) of symmetry if the line(s) exist(s) (2.4.K1f). | The student…<br>1. solves real-world problems by applying properties of plane figures (circles, squares, rectangles, triangles, ellipses) to (2.4.A1f), e.g., the teacher asked each student to draw a rectangle. A student draws a square. Did the student follow directions? Why or why not?<br>2. demonstrates how (2.4.A1f):<br>a. plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, hexagons, trapezoids) can be combined to make a new shape;<br>b. solids (cubes, rectangular prisms, cylinders, cones, spheres) can be combined to make a new shape.<br>3. identifies the plane figures (circles, squares, rectangles, triangles, ellipses, rhombi, hexagons, trapezoids) used to form a composite figure (2.4.A1f). |

**▲– Assessed Indicator on the Objective Assessment**
**■ – Assessed Indicator on the Optional Constructed Response Assessment**
**N – Noncalculator**
**($) – Financial Literacy**

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**Teacher Notes: Geometry** is the study of shapes, their properties, and their relationships to other shapes. Symbols and numbers are used to describe their properties and their relationships to other shapes. The term *geometry* comes from two Greek words meaning “earth measure.” The fundamental concepts in geometry are point (no dimension), line (one-dimensional), plane (two-dimensional), and space (three-dimensional). Plane figures are referred to as two-dimensional and solids are referred to as three-dimensional.

From the Mathematics Dictionary and Handbook (Nichols Schwartz Publishing, 1999), **property** as a mathematical term means a characteristic (an attribute) of a number, geometric shape, mathematical operation, equation, or inequality. To give an example:

- Property of a number: 8 is divisible by 2.
- Property of a geometric shape: Each of the four sides of a square is of the same length.
- Property of an operation: Addition is commutative. For all numbers x and y, $x + y = y + x$.
- Property of an equation: For all numbers a, b, and c, if $a = b$, then $a + c = b + c$.
- Property of an inequality: For all numbers a, b, and c, if $a > b$, then $a – c > b – c$.

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

Pattern blocks are a collection of six geometric shapes in six colors. Each set contains 250 pieces – 50 green triangles, 25 orange squares, 50 blue rhombi, 50 tan rhombi, 50 red trapezoids, and 25 yellow hexagons. The blue rhombus and the tan rhombus also are parallelograms, and the orange square is a parallelogram. The blocks are designed so that their sides are all the same length with the exception that the trapezoid has one side twice as long. This feature allows the blocks to be nested together and encourages the exploration of relationships among the shapes. Activities with pattern blocks help students explore patterns, functions, fractions, congruence, similarity, symmetry, perimeter, area, and graphing. A template for a pattern block set can be found in the Appendix.

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## Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

### Benchmark 2: Measurement and Estimation – The student estimates and measures using standard and nonstandard units of measure with concrete objects in a variety of situations.

| Third Grade Knowledge Base Indicators | Third Grade Application Indicators |
|---|---|
| The student…<br>1. uses whole number approximations (estimations) for length, width, weight, volume, temperature, time, and perimeter using standard and nonstandard units of measure (2.4.K1a) **($)**.<br>2. ▲ reads and tells time to the minute using analog and digital clocks (2.4.K1a).<br>3. selects, explains the selection of, and uses measurement tools, units of measure, and degree of accuracy appropriate for a given situation **to measure** (2.4.K1a) **($)**:<br>a. length width, and height to the nearest half inch, inch, foot, and yard; and to the nearest whole unit of nonstandard unit;<br>b. length, width, and height to the nearest centimeter and meter;<br>c. weight to the nearest whole unit of a nonstandard unit;<br>d. volume to the nearest cup, pint, quart, and gallon;<br>e. volume to the nearest liter;<br>f. temperature to the nearest degree.<br>4. states (2.4.K1a):<br>a. the number of hours in a day and days in a year;<br>b. the number of inches in a foot, inches in a yard, and feet in a yard;<br>c. the number of centimeters in a meter;<br>d. the number of cups in a pint, pints in a quart, and quarts in a gallon.<br>5. finds the perimeter of squares, rectangles, and triangles given the measures of all the sides (2.4.K1f). | The student…<br>1. solves real-world problems by applying appropriate measurements:<br>a. ▲ length to the nearest inch, foot, or yard, e.g., Jill has a piece of rope that is 36 inches long and Bob has a piece that is 15 inches long. If they put their pieces together, how long would the piece of rope be?<br>b. ▲ length to the nearest centimeter or meter, e.g., a new pencil is about how many centimeters long?<br>c. length to the nearest whole unit of a nonstandard unit, e.g., how many paper clips long is a hot dog?<br>d. temperature to the nearest degree, e.g., what would the temperature outside be if it was a good day for swimming?<br>e. ▲ number of days in a week, e.g., if school started 37 weeks ago, how many days of school have passed?<br>2. estimates to check whether or not measurements or calculations for length, temperature, and time in real-world problems are reasonable (2.4.A1a) **($)**, e.g., after finding the range of temperature over a two-week period, determine whether or not the answer is reasonable.<br>3. adjusts original measurement or estimation for length, weight, temperature, and time in real-world problems based on additional information (a frame of reference) (2.4.A1a) **($)**, e.g., the class estimates that the class gerbil weighs as much as a box of 24 crayons. The gerbil is placed on one side of the pan balance and a box of 16 crayons is placed on the other side. The pan balance barely moves. Should the estimate of the gerbil's weight be adjusted? |

**Teacher Notes:** The term *geometry* comes from two Greek words meaning “earth measure.” **Measurement** provides the tools required to apply geometric concepts in the real-world. **Estimation in measurement** is defined as making guesses as to the exact measurement of an object without using any type of measurement tool. Estimation helps students develop a relationship between the different sizes of units of measure. It helps students develop basic properties of measurement and it gives students a tool to determine whether a given measurement is reasonable.

**Mathematical models** such as concrete objects, pictures, diagrams, number lines, unifix cubes, hundred charts, or base ten blocks are necessary for conceptual understanding and should be used to explain computational procedures. If a mathematical model can be used to represent the concept, the indicator in the Models benchmark is identified in the parentheses. For example, (2.4.K1a) refers to Standard 2 (Algebra), Benchmark 4 (Models), and Knowledge Indicator 1a (process models). Then, the indicator in the Models benchmark lists some of the mathematical models that could be used to teach the concept. In addition, each indicator in the Models benchmark is linked back to the other indicators. Those indicators are identified in the parentheses. For example, *process models* are linked to 1.1.K3, 1.2.K6, 1.3.K1, … with 1.1.K3 referring to Standard 1 (Number and Computation), Benchmark 1 (Number Sense), and Knowledge Indicator 3.

The National Standards in **Personal Finance** identify what K-12 students should know and be able to do in personal finance; benchmarks are provided at three grade levels (grades 4, 8, and 12) and are grouped into four major categories: Income, Spending and Credit, Saving and Investing, and Money Management. Although the National Standards in Personal Finance are benchmarked at three grade levels, the indicators in the Kansas Curricular Standards for Mathematics that correlate with the National Standards in Personal Finance are indicated at each grade level with a **($)**. The National Standards in Personal Finance are included in the Appendix.

**▲– Assessed Indicator on the Objective Assessment**
**■ – Assessed Indicator on the Optional Constructed Response Assessment**
**N – Noncalculator**
**($) – Financial Literacy**

## Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

### Benchmark 3: Transformational Geometry – The student recognizes and performs one transformation on simple shapes or concrete objects in a variety of situations.

| Third Grade Knowledge Base Indicators | Third Grade Application Indicators |
|---|---|
| The student…<br>1. knows and uses cardinal points (north, south, east, west) and intermediate points (northeast, southeast, northwest, southwest) (2.4.K1a).<br>2. recognizes and performs one transformation (reflection/flip, rotation/turn, and translation/slide) on a two-dimensional figure (2.4.K1f). | The student…<br>1. recognizes real-world transformations (reflection/flip, rotation/turn, and translation/slide) (2.4.A1a), e.g., tiles in a ceiling, bricks in a sidewalk, or steps on a playground slide.<br>2. gives and uses directions to move from one location to another on a map and follows directions including the use of cardinal and intermediate points (2.4.A1a). |

**Teacher Notes: Transformational geometry** is another way to investigate geometric figures by moving every point in a plane figure to a new location. To help students form images of shapes through different transformations, students can use concrete objects, figures drawn on graph paper, mirrors or other reflective surfaces, or appropriate technology.

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## Standard 3: Geometry – The student uses geometric concepts and procedures in a variety of situations.

### Benchmark 4: Geometry From An Algebraic Perspective – The student relates geometric concepts to a number line and the first quadrant of a coordinate plane in a variety of situations.

| Third Grade Knowledge Base Indicators | Third Grade Application Indicators |
|---|---|
| The student…<br>1. uses a number line (horizontal/vertical) to model the basic multiplication facts through the 5s and the multiplication facts of the 10s (2.4.K1a).<br>2. identifies points on a coordinate plane (coordinate grid) using (2.4.K1a):<br>a. two positive whole numbers,<br>b. a letter and a positive whole number.<br>3. identifies points as ordered pairs in the first quadrant of a coordinate plane (coordinate grid) (2.4.K1a). | The student…<br>1. solves real-world problems using coordinate planes (coordinate grids) and map grids that have positive whole number and letter coordinates (2.4.A1a), e.g., identifying locations on a map or giving and following directions to move from one location to another. |

**Teacher Notes:** A **number line** (a mathematical model) is a diagram that represents numbers with equal distances marked off as points on a line, and is an example of one-to-one correspondence (a relation). A number line can be used as a visual representation of numbers and operations. In addition, a number line used horizontally and vertically is a precursor to the coordinate plane; and the distance between two numbers on a number line is a precursor to absolute value.

A **coordinate plane** (coordinate grid) consists of a horizontal number line called the x-axis and a vertical number line called the y-axis. These two lines intersect at a point called the origin. The x-axis and the y-axis divide the plane into four sections called quadrants. Any point on the coordinate plane can be named with two numbers called coordinates. The first number is the x-coordinate. The second number is the y-coordinate. Since the pair is always named in order (first x, then y), it is called an ordered pair.

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## Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.

### Benchmark 1: Probability – The student applies the concepts of probability to draw conclusions and to make predictions and decisions including the use of concrete objects in a variety of situations.

| Third Grade Knowledge Base Indicators | Third Grade Application Indicators |
|---|---|
| The student…<br>1. recognizes any outcome of a simple event in an experiment or simulation as impossible, possible, certain, likely, unlikely, or equally likely (2.4.K1g) **($)**.<br>2. ▲ ■ lists some of the possible outcomes of a simple event in an experiment or simulation including the use of concrete objects (2.4.K1g-h). | The student…<br>1. makes predictions about a simple event in an experiment or simulation; conducts the experiment or simulation including the use of concrete objects; records the results in a chart, table, or graph; and uses the results to draw conclusions about the event (2.4.A1g-h).<br>2. compares what should happen (theoretical probability/expected results) with what did happen (experimental probability/empirical results) in an experiment or simulation with a simple event (2.4.A1g). |

**▲– Assessed Indicator on the Objective Assessment**
**■ – Assessed Indicator on the Optional Constructed Response Assessment**
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**Teacher Notes:** Ideas from **probability** reinforce concepts in the other Standards, especially Number and Computation and Geometry. Students need to develop an intuitive concept of chance – whether or not something is unlikely or likely to happen. Probability experiences should be addressed through the use of concrete objects, coins, and geometric models (spinners, number cubes, or dartboards). Probabilities are ratios, expressed as fractions, decimals, or percents, determined by considering results or outcomes of experiments. Some examples of uses of probability in every day life include: There is a 50% chance of rain today. What is the probability that the team will win every game?

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## Standard 4: Data – The student uses concepts and procedures of data analysis in a variety of situations.

### Benchmark 2: Statistics – The student collects, organizes, displays, explains, and interprets numerical (whole numbers) and non-numerical data sets including the use of concrete objects in a variety of situations.

| Third Grade Knowledge Base Indicators | Third Grade Application Indicators |
|---|---|
| The student…<br>1. organizes, displays, and reads numerical (quantitative) and non-numerical (qualitative) data in a clear, organized, and accurate manner including a title, labels, categories, and whole number intervals using these **data displays** (2.4.K1h) **($)**:<br>a. graphs using concrete objects;<br>b. pictographs with a whole symbol or picture representing one, two, five, ten, twenty-five, or one-hundred (no partial symbols or pictures);<br>c. frequency tables (tally marks);<br>d. horizontal and vertical bar graphs;<br>e. Venn diagrams or other pictorial displays, e.g., glyphs;<br>f. line plots;<br>g. charts and tables.<br>2. collects data using different techniques (observations, polls, surveys, or interviews) and explains the results (2.4.K1h) **($)**.<br>3. ▲ finds these statistical measures of a data set with less than ten data points using whole numbers from 0 through 1,000 (2.4.K1a) **($)**:<br>a. minimum and maximum data values,<br>b. range,<br>c. mode (uni-modal only),<br>d. median when data set has an odd number of data points. | The student…<br>1. interprets and uses data to make reasonable inferences and predictions, answer questions, and make decisions from these **data displays** (2.4.A1h) **($)**:<br>a. graphs using concrete objects;<br>b. pictographs with a whole symbol or picture representing one, two, five, ten, twenty-five, or one-hundred (no partial symbols or pictures);<br>c. frequency tables (tally marks);<br>d. horizontal and vertical bar graphs;<br>e. Venn diagrams or other pictorial displays;<br>f. line plots;<br>g. charts and tables.<br>2. uses these statistical measures with a data set of less than ten data points using whole numbers from 0 through 1,000 to make reasonable inferences and predictions, answer questions, and make decisions (2.4.A1a) **($)**:<br>a. minimum and maximum data values,<br>b. range,<br>c. mode,<br>d. median when data set has an odd number of data points.<br>3. recognizes that the same data set can be displayed in various formats including the use of concrete objects (2.4.A1h) **($)**. |

**Teacher Notes: Graphs (data displays)** are pictorial representations of mathematical relationships, are used to tell a story, and are an important part of statistics. When a graph is made, the axes and the scale (numbers running along a side of the graph) are chosen for a reason. The difference between numbers from one grid line to another is the **interval.** The interval will depend on the lowest and highest values in the data set. Emphasizing the importance of using equal-sized pictures or intervals is critical to ensuring that the data display is accurate.

Graphs take many forms:
- bar graphs and pictographs compare discrete data,
- frequency tables show how many times a certain piece of data occurs,
- circle graphs (pie charts) model parts of a whole,
- line graphs show change over time,
- Venn diagrams show relationships between sets of objects, and
- line plots show frequency of data on a number line.

An important aspect of data is its *center.* The **measures of central tendency** (averages) of a data set are mean, median, and mode. Conceptual understanding of mean, median, and mode is developed through the use of concrete objects that represent the data values.

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